# Creative Contests - Theory and Experiment 

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#### Abstract

In many competitions where creativity and innovation play a large role (e.g., architecture design competitions or research grant competitions), contestants can be uncertain about the organizer's exact preferences. I develop a model of creative contests in which two firms compete by adjusting their designs when they are uncertain about the contest organizer's ideal design. My model contrasts with existing contest models, as the latter assume organizer preferences instead to be public knowledge. A model of creative contests that accounts for such uncertainty enables us to study many new questions. In particular, I investigate whether an organizer should disclose her ideal design to contestants and find that disclosure is not always optimal for organizers, because disclosing an ideal design favors one participant over others and thus discourages competition. I also conduct a laboratory experiment to test the model's empirical relevance when assumptions about rationality and risk-neutrality are not necessarily satisfied and find that the results are generally consistent with theoretical predictions for contestants' behavior and for whether the organizer benefits from disclosure.


Keywords: all-pay contests, uncertainty, information disclosure, experiment

## 1 Introduction

Contests are widely used to allocate scarce resources. In many contests, such as lobbying, college admissions, and sports competitions, it is clear to the contestants what the contest organizer asks for and how she will rank their performance. This is less clear in other contests, such as architecture design competitions and research grant competitions. I refer to such competitions as creative contests.

The following example illustrates the key feature of creative contests: in 1980, the Vietnam Veterans Memorial Design Competition was organized in the US. At the time that contestants submitted their designs, the judges' precise preferences regarding various possible designs were not known to the contestants. Indeed, the winner, Maya Ying Lin, was surprised because her design had earned her a B in a class assignment.

Competitions in which the ranking of contestants' possible submissions is publicly known at the outset of the competition have been widely studied in the contest literature [see Konrad (2009) and Vojnović (2016) for a review]. In such contests, contestants make tradeoffs between two forces: more effort increases the chance of winning but also costs more. Meanwhile, some contest models include uncertainty or noises, such as in Tullock (1980) lottery contests and Lazear and Rosen (1981) type tournaments. Noise and uncertainty lead to a probabilistic prize allocation, but it is publicly known that the contest organizer prefers higher effort to lower effort on average anyway.

This paper, thus, develops a simple two-player model that accommodates a key feature of creative contests: the fact that contestants are uncertain about the organizer's exact preferences. Specifically, in addition to the trade-off mentioned above between the chance of winning and the costs of effort, contestants in the new model also take into account their uncertainty regarding what the organizer is asking for.

My model consists of one contest organizer and two design firms. The design firms are symmetrically uninformed about the organizer's ideal design and compete for a prize by
each submitting a design. After both firms submit their designs, the organizer ranks the designs based on how close they are to her ideal. Each firm has a different initial design, which can be adjusted at a cost. Moreover, the firms may value the prize differently.

I characterize the unique Nash equilibrium and then consider the following thought experiment. If the contest organizer has the option to disclose her ideal design to the firms, does it always benefit her to do so? I find that disclosing the ideal design is, in fact, not always optimal. When firms' adjustment costs are low, the organizer does benefit from disclosing because both firms can easily adjust their designs; as a result, the winning design is closer to the organizer's ideal. However, there is also a countervailing force: when the ideal design is disclosed, the firm whose initial design is closer to that ideal gains an advantage. This fact discourages its opponent from competing further - which in turn reduces the incentives of the advantaged firm to compete further as well. This discouragement effect increases as adjustment costs increase, because it grows harder for the disadvantaged firm to catch up. When adjustment costs are high enough, then, the contest organizer is made worse off by disclosing her ideal design.

My model assumes rationality and risk neutrality. However, as these conditions are not necessarily satisfied in real-life situations, I conduct a laboratory experiment to test the model's empirical relevance. I frame the experiment using a Hotelling (1929) model, in which two firms make costly moving decisions. After both firms make their decisions, then, a computerized consumer arrives at a random location and makes a purchase. The experimental results broadly support my model's predictions relating to firms' behavior, as well as the comparative static prediction regarding the benefit of disclosure; in particular, subjects compete more aggressively when moving costs are low and when the consumer is disclosed to be near the midpoint between the two firms' initial locations. The consumer's welfare, measured as the distance from the winning firm, is significantly improved when moving costs are low, but insignificantly improved when moving costs are high.

The contribution of this paper is threefold. First, I develop a model that incorporates
contestants' uncertainty about the organizer's objective. The closest paper in this respect is Letina and Schmutzler (2017), in which a buyer induces innovations from two suppliers and in which the ideal approach is unknown to both the buyer and the suppliers. In Letina and Schmutzler (2017), all approaches have the same cost, and the authors consider how the buyer (organizer) could induce various approaches by designing a mechanism with allowable bid prices and transfers. My model considers a given mechanism and allows contestants to have different and non-constant cost functions.

The second contribution is that it studies an information disclosure problem in all-pay contests. Some existing papers have looked at a simpler version of the question in different settings. For example, Kaplan (2012) considers whether a buyer would be better off by communicating her preferences to sellers. A design with a preferred style generates the same premium to the buyer, and styles could either be left unchanged or could be changed at zero cost. My model allows the organizer to value styles differently and allows contestants to change their designs with a continuous cost function; thus, the model captures more of the elements one might find in the field.

Lastly, this paper also contributes to the literature on contest experiments; my experiment allows us to examine directly how and whether information affects behaviors in a contest experiment. I find that, consistent with overbidding behavior found in the literature [Sheremeta (2013) and Gneezy and Smorodinsky (2006)], subjects overbid in the experiment. ${ }^{1}$ Specifically, I find that overbidding is reduced when consumer locations are disclosed, and it is reduced further when subjects are disclosed to be at a disadvantage within the competition. This suggests that having an advantageous position and incomplete information about the consumer location (or, in the current study, ideal design) may be additional sources of overbidding behavior, beyond the various sources of such behavior summarized in Sheremeta (2013).

The remainder of this paper is organized as follows. Section 2 sets up the model. Section

[^0]3 solves for the unique Nash equilibrium. Section 4 investigates the organizer's information disclosure problem. Section 5 empirically tests theoretical predictions about contestants' behavior and the contest organizer's welfare in a laboratory experiment. Section 6 discusses a generalization of the model and its connections with the literature from other fields. Section 7 concludes. Proofs not in the text appear in the appendix.

## 2 The Model

A risk-neutral contest organizer hosts an architectural design competition to elicit a design for a new building. Let $\Omega=[0,1]$ be the set of all possible designs. Two risk-neutral design firms, labeled by $A$ and $B$, compete for a single prize by submitting designs $a, b \in \Omega$ simultaneously and independently. ${ }^{2}$ Denote Firm $i=A, B$ 's valuation of the prize by $V_{i}$. Firm $A$ has an initial design $\alpha_{0}$ and Firm $B$ has an initial design $\beta_{0}$. Firm $i=A, B$ incurs a sunk $\operatorname{cost} c_{i}$ that depends on the adjustment of the submitted design relative to its initial design.

After receiving both submissions, the organizer rank the designs based on their distances from her ideal design $s^{*}$. I assume that $s^{*}$ is randomly drawn from a uniform distribution $U[0,1]$. The firm whose submission is closer to $s^{*}$ wins the prize. In case of a tie, both firms have equal probabilities of winning. Therefore, when submissions are ( $a, b$ ), firms' expected utilities are

$$
\begin{equation*}
U_{A}(a, b)=P_{A}(a, b) V_{A}-c_{A}\left(\left|a-\alpha_{0}\right|\right) \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
U_{B}(a, b)=P_{B}(a, b) V_{B}-c_{B}\left(\left|b-\beta_{0}\right|\right), \tag{2}
\end{equation*}
$$

[^1]where expected probability of winning $P_{A}$ and $P_{A}$ satisfy:
\[

1-P_{B}(a, b)=P_{A}(a, b)= $$
\begin{cases}\frac{a+b}{2} & \text { if } a<b \\ \frac{1}{2} & \text { if } a=b \\ 1-\frac{a+b}{2} & \text { if } a<b\end{cases}
$$
\]

Figure 1 below graphically illustrates the model.
Figure 1: illustration of the model


I consider creative contests that satisfy the following assumption:

Assumption 1. $c_{i}$ is quadratic.

This assumption, together with the uniform distribution, implies that expected utilities in (1) and (2) are strictly concave. This guarantees the uniqueness of pure-strategy Nash equilibria whenever such an equilibrium exists. When there exists only a mixed-strategy Nash equilibrium, meanwhile, strict concavity guarantees a nice structure of its support. Assumption 1 can be generalized to any increasing, twice differentiable, and strictly convex costs; this assumption is common in the contest literature. More details can be found in Section 6.2.

For notational simplicity, I focus on the case where two firms' initial designs are extreme: $\alpha_{0}=0$ and $\beta_{0}=1$. Recall that firms could be asymmetric in two aspects: adjustment costs and initial designs. Hence, for instance, one firm may have an advantage in the competition over the other firm because it bears a lower cost to make the same adjustments. Alternatively, a firm may be at an advantage because its initial design is more "mainstream" (i.e. close to
the median of the distribution of possible designs) and therefore needs fewer adjustments in the first place. The ways in which these two sources of asymmetry interact and affect the organizer's welfare are not the primary focus of the current paper, but interested readers can refer to Zhu (2019).

## 3 Solving For Equilibrium

### 3.1 Existence

A Nash equilibrium is a strategy profile in which each firm's strategy assigns probability one to the firm's best response set, given the other firm's strategy. The existence of Nash equilibria in creative contests is not obvious because utility functions are discontinuous. When both firms choose the same design $s$, a slight deviation to its left or right yields winning probabilities of $s$ and $1-s$, which are not equal unless $s=\frac{1}{2}$. This discontinuity may lead to non-existence of Nash equilibrium; see Sion and Wolfe (1957) and Shaked (1975). I apply Dasgupta and Maskin (1986)'s existence result to prove that there exists a Nash equilibrium in every creative contest.

Proposition 1. Every creative contest has a Nash equilibrium.

The equilibrium could be either in pure or mixed strategies. In what follows, I first provide a necessary and sufficient condition for the existence of a pure strategy Nash equilibrium (PSNE henceforth), as well as a characterization of the equilibrium. I then examine cases in which PSNE does not exist and solve for a mixed strategy Nash equilibrium (MSNE henceforth), which is bound to exist by Proposition 1. At the end of Section 3, I show that the characterized equilibrium, either in pure or mixed strategies, is the unique equilibrium.

### 3.2 Pure Strategy Nash Equilibrium

To characterize PSNE, I introduce the following definition.

Definition 1. Firm i's left (right) optimal design is its best response when the other firm chooses the rightmost (leftmost) design.

As an example, firm $A$ 's left optimal design, denoted by $\tilde{a}_{L}$, maximizes $U_{A}(a, 1)=\frac{a+1}{2}-c_{A}(a)$; likewise, firm B's right optimal submission $\tilde{b}_{R}$ maximizes $U_{B}(0, b)=1-\frac{b}{2}-c_{B}(1-b)$.

This definition is useful in characterizing PSNE because $\tilde{a}_{L}$ and $\tilde{b}_{R}$ depend solely on the model primitives but capture firm $i$ 's local incentive to deviate. Figure 2 illustrates how $\tilde{a}_{L}$ can be used to determine whether $a$ is a best response to $b$. The plotted curve is firm $A$ 's expected utility from submitting $a$ when $B$ chooses 1: $U_{A}(a, 1)=\frac{a+1}{2}-c_{A}(a)$, which attains its maximum at $\tilde{a}_{L}$. For any $a \in[0, b)$, firm $A$ 's expected utility for submission $(a, b)$ differs from $U_{A}(a, 1)$ by a constant because $U_{A}(a, b)=\frac{a+b}{2}-c_{A}(a)=U_{A}(a, 1)-\frac{1-b}{2}$. In other words, $U_{A}(a, b)$ also reaches its maximum at $\tilde{a}_{L}$ for $a \in[0, b)$. Hence $a$ is not a best response to $b$.

Figure 2: illustration of $\tilde{a}_{L}$ and its use


More formally, Definition 1 can pin down the unique candidate of PSNE, as Claim 1 states. Claim 1. If $\tilde{a}_{L}<\tilde{b}_{R}$, the unique candidate of PSNE is $\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$; otherwise, it is $\left(\frac{1}{2}, \frac{1}{2}\right)$.

The intuition is the following. It cannot be the case where $(a, b)$ is an equilibrium with $a>b$ because firm $A$ would deviate by reducing the adjustment and submitting a design that is to the immediate right of $b$. If $(a, b)$ is an equilibrium where $a=b$, it must be $a=b=\frac{1}{2}$
because otherwise probabilities for $s^{*}$ to fall on both sides of $a=b$ are not equal and both firms would "undercut" their opponent to obtain the larger probability of winning. If $(a, b)$ is an equilibrium where $a<b$, from the explanations above, it must be $a=\tilde{a}_{L}$ and $b=\tilde{b}_{R}$; otherwise, it is not a mutually best response.

The conditions in Claim 1 are necessary but not sufficient. Figure 3 illustrates the reason and adds additional necessary conditions for $\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$ to be an equilibrium. In the figure, $\tilde{a}_{L}$ and $\tilde{b}_{R}$ are so close so that when firm $A$ could gain by deviating from $\tilde{a}_{L}$ to the immediate right of $\tilde{b}_{R}$. This means, for $\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$ to be an equilibrium, besides $a=\tilde{a}_{L}$ and $b=\tilde{b}_{R}$, it must be $U_{A}\left(\tilde{a}_{L}, \tilde{b}_{R}\right) \geq \lim _{a \downarrow \tilde{b}_{R}} U_{A}\left(a, \tilde{b}_{R}\right)$ and $U_{B}\left(\tilde{a}_{L}, \tilde{b}_{R}\right) \geq \lim _{b \uparrow \tilde{a}_{L}} U_{B}\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$.

Figure 3: an additional constraint for $(\tilde{a}, \tilde{b})$ to be PSNE


It turns out these two sets of constraints together are sufficient for PSNE. Theorem 1 summarizes necessary and sufficient conditions for PSNE and is the main result of this section.

Theorem 1. The unique PSNE is $\left(\frac{1}{2}, \frac{1}{2}\right)$ if and only if $\tilde{a}_{L} \geq \frac{1}{2} \geq \tilde{b}_{R}$. The unique PSNE is $\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$ if and only if $\tilde{a}_{L}<\tilde{b}_{R}, U_{A}\left(\tilde{a}_{L}, \tilde{b}_{R}\right) \geq \lim _{a \downarrow \tilde{b}_{R}} U_{A}\left(a, \tilde{b}_{R}\right)$ and $U_{B}\left(\tilde{a}_{L}, \tilde{b}_{R}\right) \geq \lim _{b \uparrow \tilde{a}_{L}} U_{B}\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$. In other cases, there exists no PSNE.

Proof. $\Longrightarrow:$ By Claim 1, when $\left(\frac{1}{2}, \frac{1}{2}\right)$ is a PSNE, it must be $\tilde{a}_{L}>\tilde{b}_{R}$. If $\tilde{a}_{L}<\frac{1}{2}$, by definition of $\tilde{a}_{L}, U_{A}\left(\tilde{a}_{L}, \frac{1}{2}\right)>U_{A}\left(\frac{1}{2}, \frac{1}{2}\right)$. Hence $\left(\frac{1}{2}, \frac{1}{2}\right)$ is not an equilibrium. This implies $\tilde{a}_{L} \geq \frac{1}{2}$. Similar logic applies to $\tilde{b}_{R} \leq \frac{1}{2}$.

When $\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$ is an equilibrium, the last two conditions must hold by definition of a Nash equilibrium, while the first follows from Lemma 1.
$\Longleftarrow$ : uniqueness follows from Lemma 1. To show they are indeed equilibria when conditions are satisfied: when $\tilde{a}_{L} \geq \frac{1}{2}$, firm $A$ has no profitable deviations from $\frac{1}{2}$ to $a \in\left[0, \frac{1}{2}\right]$ because of
the definition of $\tilde{a}_{L}$. Firm $A$ also has no incentive to deviate to $\left(\frac{1}{2}, 1\right)$ because it increases costs but lowers the winning probability. Similar argument shows Firm $B$ also has no incentive to deviate from $\frac{1}{2}$.

When $\tilde{a}_{L}<\tilde{b}_{R}$, by definition of $\tilde{a}_{L}, \tilde{a}_{L}$ is a best response to $\tilde{b}_{R}$ for $a \in\left[0, \tilde{b}_{R}\right)$. Because $U_{A}\left(\tilde{a}_{L}, \tilde{b}_{R}\right) \geq \lim _{a \downarrow \tilde{b}_{R}} U_{A}\left(a, \tilde{b}_{R}\right)$, Firm $A$ also has no incentive to deviate to any design on $\left(\tilde{b}_{R}, 1\right]$. This shows that firm $A$ has no incentive to deviate. Similar argument shows firm $B$ has no incentive to deviate from $\tilde{b}_{R}$.

Theorem 1 suggests that adjustment costs determine Nash equilibria in the following way. When both firms have low marginal costs, adjustment costs are low and $\tilde{a}_{L}$ and $\tilde{b}_{R}$ are both far away from firms' initial designs, 0 and 1 , implying $\tilde{a}_{L}>\tilde{b}_{R}$. The only "agreement" they could achieve is for both firms to choose $\frac{1}{2}$; Hotelling location model is a limit case, in which marginal costs are both zero. On the other hand, when marginal costs are high, adjustment is costly for both firms. $\tilde{a}_{L}$ and $\tilde{b}_{R}$ are hence close to firms' featured designs, 0 and 1 , implying $\tilde{a}_{L}<\tilde{b}_{R}$. Therefore, in equilibrium, they both make small adjustments towards each other. When firms are asymmetric in adjustment costs, there might be no PSNE.

A special case arises when both firms have the same adjustment cost. I call such contests symmetric contests. In this case, a pure strategy equilibrium always exists.

Proposition 2. Symmetric creative contests have a unique PSNE.

Proof. When $c_{A}=c_{B}, \tilde{a}_{L}+\tilde{b}_{R}=1$. If $\tilde{a}_{L}<\tilde{b}_{R}$, by symmetry, it must be $\tilde{a}_{L}<\frac{1}{2}<\tilde{b}_{R}$. Hence firm $A$ has no incentive to deviate to the immediate right of $\tilde{b}_{R}$. Similarly, firm $B$ has no incentive to deviate to the immediate left of $\tilde{a}_{L}$. According to Theorem 1 , the unique $\operatorname{PSNE}$ is $\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$.

If $\tilde{a}_{L}>\tilde{b}_{R}$, by symmetry, it must be $\tilde{a}_{L}>\frac{1}{2}>\tilde{b}_{R}$. According to Theorem 1 , the unique PSNE is $\left(\frac{1}{2}, \frac{1}{2}\right)$.

### 3.3 Mixed Strategy Nash Equilibrium

Mixed strategy Nash equilibria can be pinned down once firms' best response sets are identified. Instead of working with best response sets directly, I focus my attention on their support, defined as follows.

Definition 2. The support of a strategy is the smallest closed set to which the strategy assigns probability one.

The following claim is useful in identifying the supports of the equilibrium strategies.

Claim 2. In equilibrium, any accumulation point $s$ in the support is a best response if the opponent does not have an atom at $s$.

Proof. This is an implication of continuity. When firm $j$ has no atom at $s$, firm $i$ 's expected utility is continuous at $s$. When $s$ is an accumulation point in firm $i$ 's support, there is a sequence of points approaching $s$ in his best response set. firm $i$ gets his equilibrium payoffs at all these points, and by continuity, he also gets his equilibrium payoffs at $s$.

In the remaining of the paper, I will focus on regular equilibria, defined as follows. ${ }^{3}$

Definition 3. A strategy is regular if its support is as a finite union of closed intervals. A mixed strategy Nash equilibrium is regular if both firms' strategies are regular.

Proposition 3 below characterizes the support of equilibrium strategies.

Proposition 3. In equilibrium, both firms continuously randomize on a common interval $[\alpha, \beta]$. Moreover, each firm has at most one atom:

1. If $\tilde{a}_{L}, \tilde{b}_{R}$ are both smaller than $\frac{1}{2}$, firm $A$ 's atom is $\tilde{a}_{L}$ and firm $B$ 's atom is $\alpha$, satisfying $\tilde{a}_{L} \leq \alpha<\beta<\frac{1}{2}$ and $\tilde{b}_{R}<\beta$.

[^2]2. If $\tilde{a}_{L}, \tilde{b}_{R}$ are both greater than $\frac{1}{2}$, firm $A$ 's atom is $\beta$ and firm $B$ 's atom is $\tilde{b}_{R}$, satisfying $\frac{1}{2}<\alpha<\beta \leq \tilde{b}_{R}$ and $\tilde{a}_{L}>\alpha$.

Proposition 3 is proved using the following 4 lemmas.
Lemma 1. In equilibrium, if firm $i$ best responds at two points $l<r$, firm $j$ must choose $[l, r]$ with positive probability.

Proof. Suppose otherwise: given firm $j$ 's strategy $G_{j}$, firm $i$ obtains her maximum expected utility at both $l$ and $r$; however, firm $j$ choose $[l, r]$ with zero probability.

Expected utility for firm $i$ to choose $l$ is

$$
U_{i}(l)=\int_{0}^{l}\left(1-\frac{z+l}{2}\right) d G_{j}(z)+\int_{r}^{1}\left(\frac{z+l}{2}\right) d G_{j}(z)-c_{i}(l)
$$

Similarly, expected utility for firm $i$ to choose $r$ is

$$
U_{i}(r)=\int_{0}^{l}\left(1-\frac{z+r}{2}\right) d G_{j}(z)+\int_{r}^{1}\left(\frac{z+r}{2}\right) d G_{j}(z)-c_{i}(r)
$$

Moreover, because firm $j$ chooses $[l, r]$ with zero probability, expected utility for firm $i$ to choose any $s \in[l, r]$ is

$$
\begin{aligned}
U_{i}(s) & =\int_{0}^{l}\left(1-\frac{z+s}{2}\right) d z+\int_{r}^{1}\left(\frac{z+s}{2}\right) d z-c_{i}(s) \\
& =C+\frac{s}{2}\left(1-2 G_{j}(l)\right)-c_{i}(s)
\end{aligned}
$$

where $C=\int_{0}^{l}\left(1-\frac{z}{2}\right) d G_{j}(z)+\int_{r}^{1} \frac{z}{2} d G_{j}(z)$ does not depend on $s$. By Assumptions ?? and 1, $U_{i}(s)$ is strictly concave in $s$. This contradicts the statement that $l$ and $r$ both are in the best response set.

Lemma 1 does not rule out gaps in the support. For example, suppose firm 1 has an atom at $s=0$ and continuously randomizes on $[\alpha, \beta]$; firm 2 has an atom at $\alpha$ and continuously
randomizes on $[\alpha, \beta]$. This structure does not violate Lemma 1, but firm1's support contains a gap $(0, \alpha)$.

Despite this, Lemma 1 implies the following two lemmas, which inspect intervals and atoms separately.

Lemma 2 (One Interval). In equilibrium, both firms randomize on one common interval $[\alpha, \beta]$.

I first show that firms randomize on some common intervals, after which I prove there must be only one interval. Below is a sketch of the proof.

Suppose firms randomize on different intervals: there exists some interval $[l, r]$ such that it is in firm $i$ 's support but not in firm $j$ 's. In other words, firm $i$ best responds at both $l$ and $r$, but firm $j$ chooses $[l, r]$ with zero probability. This contradicts Lemma 1.

The intervals the firms randomize on must be one interval. For simplicity, let us assume no atoms in the support. The complete analysis that takes potential atoms into considerations is in the appendix. If there is a gap in the intervals, two endpoints of the gap will form a $[l, r]$ such that one firm does not put any probability on it, yet the other firm best responds at both $l$ and $r$. This again contradicts Lemma 1 .

Lemma 3 shows that there cannot be more than one atom for each player.

Lemma 3 (One Atom). In equilibrium, each firm has at most one atom in the support.

Again for simplicity, in this sketch of proof let us ignore the common interval and focus on atoms. The complete proof is in the appendix. Suppose that firm $A$ has two atoms $a_{1}<a_{2}$. Notice that the marginal benefit at these two points satisfy $M B_{1} \leq M B_{2}$, but the marginal costs are ranked $M C_{1}>M C_{2}$. Therefore, it cannot be the case that both $a_{1}$ and $a_{2}$ are optimal for firm $A$.

Lastly, the following lemma provides the last piece of characterization of support.

Lemma 4 (Relationship). $\tilde{a}_{L}, \tilde{a}_{L}$ and the common interval $[\alpha, \beta]$ satisfies the following relationship:

If $\tilde{a}_{L}, \tilde{b}_{R}$ are both smaller than $\frac{1}{2}$, firm $A$ 's atom is $\tilde{a}_{L}$ and firm $B$ 's atom is $\alpha$; moreover, $\tilde{a}_{L} \leq \alpha<\beta<\frac{1}{2}$ and $\tilde{b}_{R}<\beta$.

If $\tilde{a}_{L}, \tilde{b}_{R}$ are both greater than $\frac{1}{2}$, firm $A$ 's atom is $\beta$ and firm $B$ 's atom is $\tilde{b}_{R}$; moreover, $\frac{1}{2}<\alpha<\beta \leq \tilde{b}_{R}$ and $\tilde{a}_{L}>\alpha$.

Proposition 3 characterizes the structure of support of mixed strategy equilibrium. Based on that, the algorithm proposed below determines the exact $[\alpha, \beta]$, sizes of atoms, and how each firm randomizes over this interval.

## An algorithm to find MSNE:

Step 1. Compute $\tilde{a}_{L}$ and $\tilde{b}_{R}$. Thereom 1 tells whether there is PSNE. If there is no PSNE, one of the following two cases happens:

Case 1: $\tilde{a}_{L}<\frac{1}{2}$ and $\tilde{b}_{R}<\frac{1}{2}$. According to Proposition 3, firm $A$ 's support is $\tilde{a} \cup[\alpha, \beta]$ and firm $B$ 's support is $[\alpha, \beta]$. Firm $A$ has an atom of size $p_{A}$ at $\tilde{a}_{L}$ and firm $B$ has an atom of size $p_{B}$ at $\alpha$.

Step 2: On $[\alpha, \beta]$, each firm's equilibrium strategy is characterized by the solution to a system first-order differential equation. Specifically,

$$
\begin{aligned}
& U_{A}(z)=\int_{\alpha}^{z}\left[1-\frac{z+x}{2}\right] g_{B}(x) d x+\int_{z}^{\beta}\left[\frac{z+x}{2}\right] g_{B}(x) d x-c_{A}(z)+p_{B} *\left[1-\frac{z+\alpha}{2}\right], \\
& U_{B}(z)=\int_{\alpha}^{z}\left[1-\frac{z+x}{2}\right] g_{A}(x) d x+\int_{z}^{\beta}\left[\frac{z+x}{2}\right] g_{A}(x) d x-c_{B}(1-z)+p_{A} *\left[1-\frac{z+\tilde{a}_{L}}{2}\right] .
\end{aligned}
$$

Corresponding first-order conditions are:

$$
\begin{aligned}
& U_{A}^{\prime}(z)=g_{B}(z)(1-2 z)+\frac{1}{2}-G_{B}(z)-c_{A}^{\prime}(z)-p_{B}=0 \\
& U_{B}^{\prime}(z)=g_{A}(z)(1-2 z)+\frac{1}{2}-G_{A}(z)+c_{B}^{\prime}(1-z)-p_{A}=0
\end{aligned}
$$

where $G_{i}\left(z ; \lambda_{i}\right)$ is the probability of choosing a bid on $[\alpha, z]$, and $g_{i}(z)$ is the corresponding probability distribution function.

Step 3: There are 6 unknowns in total: $\lambda_{A}$ and $\lambda_{B}$ are the constant in the solution to the ordinary differential equations (ODE). $p_{A}$ and $p_{B}$ are sizes of atoms for each player, $\alpha$ and $\beta$ are the location of the common interval. They are simultaneously characterized by the solution to the following equation system consisting of 6 equations, 4 of which are derived from boundary conditions of the ODE:

$$
\left\{\begin{array}{l}
G_{A}(\alpha)=0  \tag{3}\\
G_{A}(\beta)=1-p_{A} \\
G_{B}(\alpha)=0 \\
G_{B}(\alpha)=1-p_{B}
\end{array}\right.
$$

The rest 2 equations are about the sizes of atoms: $\tilde{a}$ is optimal for firm $A$ implies that

$$
\begin{equation*}
U_{A}\left(\tilde{a}_{L}\right)=\lim _{a \downarrow \alpha} U_{A}(a) \tag{4}
\end{equation*}
$$

$\alpha$ is optimal for firm $B$ implies that

$$
\begin{equation*}
\lim _{b \uparrow \alpha} U_{B}^{\prime}(b)=0 . \tag{5}
\end{equation*}
$$

Case 2: $\tilde{a}_{L}>\frac{1}{2}$ and $\tilde{b}_{R}>\frac{1}{2}$. According to Proposition 3, firm $A$ 's support is $[\alpha, \beta]$ and firm $B$ 's support is $[\alpha, \beta] \cup \tilde{b}_{R}$. firm $A$ has an atom of size $p_{A}$ at $\beta$ and firm $B$ has an atom of size $p_{B}$ at $\tilde{b}_{R}$.

Step 2: On $[\alpha, \beta]$, each firm's equilibrium strategy is characterized by the solution to a
system first-order differential equation. Specifically,

$$
\begin{aligned}
& U_{A}(z)=\int_{\alpha}^{z}\left[1-\frac{z+x}{2}\right] g_{B}(x) d x+\int_{z}^{\beta}\left[\frac{z+x}{2}\right] g_{B}(x) d x-c_{A}(z)+p_{B} *\left[\frac{z+\tilde{b}_{R}}{2}\right], \\
& U_{B}(z)=\int_{\alpha}^{z}\left[1-\frac{z+x}{2}\right] g_{A}(x) d x+\int_{z}^{\beta}\left[\frac{z+x}{2}\right] g_{A}(x) d x-c_{B}(1-z)+p_{A} *\left[\frac{z+\beta}{2}\right] .
\end{aligned}
$$

Corresponding first-order conditions are:

$$
\begin{aligned}
& U_{A}^{\prime}(z)=g_{B}(z)(1-2 z)+\frac{1}{2}-G_{B}(z)-c_{A}^{\prime}(z)=0 \\
& U_{B}^{\prime}(z)=g_{A}(z)(1-2 z)+\frac{1}{2}-G_{A}(z)+c_{B}^{\prime}(1-z)=0
\end{aligned}
$$

where $G_{i}\left(z ; \lambda_{i}\right)$ is the probability of choosing a bid on $[\alpha, z]$, and $g_{i}(z)$ is the corresponding probability distribution function.

Step 3: There are 6 unknowns. $\lambda_{A}$ and $\lambda_{B}$ are the constant in the solution to ODEs. $p_{A}$ and $p_{B}$ are sizes of atoms for each player, $\alpha$ and $\beta$ are the location parameters of the common interval. They are simultaneously characterized by the solution to the following equation system consisting of 6 equations, 4 of which are derived from boundary conditions of the ODE:

$$
\left\{\begin{array}{l}
G_{A}(\alpha)=0 \\
G_{A}(\beta)=1-p_{A} \\
G_{B}(\alpha)=0 \\
G_{B}(\alpha)=1-p_{B}
\end{array}\right.
$$

The rest 2 equations are about the sizes of atoms: $\tilde{b}_{R}$ is optimal for firm $B$ implies that $U_{B}\left(\tilde{b}_{R}\right)=\lim _{b \uparrow \beta} U_{B}(b) . \beta$ is optimal for firm $A$ implies that $\lim _{a \downarrow \beta} U_{A}^{\prime}(a)=0$.

The following example illustrates how the algorithm works.
Example: $c_{A}(d)=d^{2}, c_{B}(d)=\frac{1}{4} d^{2}$.
Step 1: According to the definition, $\tilde{a}=\frac{1}{4}$ and $\tilde{b}=0$. Since $\tilde{b}<\tilde{a}<\frac{1}{2}$, proceed as case 1.

Step 2: At any point $z \in[\alpha, \beta]$, expected utility $U_{i}(z)$ of firm $i$, given strategy of firm $j$, are the following:

$$
\begin{aligned}
& U_{A}(z)=\int_{\alpha}^{z}\left[1-\frac{z+x}{2}\right] g_{B}(x) d x+\int_{z}^{\beta}\left[\frac{z+x}{2}\right] g_{B}(x) d x-c_{A}(z)+p_{B} *\left[1-\frac{z+\alpha}{2}\right] \\
& U_{B}(z)=\int_{\alpha}^{z}\left[1-\frac{z+x}{2}\right] g_{A}(x) d x+\int_{z}^{\beta}\left[\frac{z+x}{2}\right] g_{A}(x) d x-c_{B}(1-z)+p_{A} *\left[1-\frac{z+\tilde{a}}{2}\right]
\end{aligned}
$$

First order conditions are:

$$
\begin{align*}
& U_{A}^{\prime}(z)=g_{B}(z)(1-2 z)+\frac{1}{2}-G_{B}(z)-2 z-p_{B}=0  \tag{6}\\
& U_{B}^{\prime}(z)=g_{A}(z)(1-2 z)+\frac{1}{2}-G_{A}(z)+\frac{1}{2}(1-z)-p_{A}=0 \tag{7}
\end{align*}
$$

where $G_{i}(z)$ is the probability of choosing a bid on $[0, z]$, and $g_{i}(z)$ is the corresponding probability distribution function. Solutions to these differential equations, namely equilibrium strategies, are:

$$
\begin{aligned}
G_{A}(z) & =\frac{\lambda_{A}}{\sqrt{1-2 z}}+\frac{1}{6} z+\frac{1}{6}-p_{A} \\
G_{B}(z) & =\frac{\lambda_{B}}{\sqrt{1-2 z}}-\frac{2}{3} z-\frac{1}{6}-p_{B}
\end{aligned}
$$

, where $\lambda_{i}$ is the constant in solutions to ODEs.
Step 3: All six unknowns $\left(\alpha, \beta, p_{A}, p_{B}, \lambda_{A}, \lambda_{B}\right)$ are determined by two sets of conditions. The first set of condition is boundary conditions:

$$
\left\{\begin{array} { l } 
{ G _ { A } ( \alpha ) = 0 }  \tag{8}\\
{ G _ { A } ( \beta ) = 1 - p _ { A } } \\
{ G _ { B } ( \alpha ) = 0 } \\
{ G _ { B } ( \beta ) = 1 - p _ { B } }
\end{array} \Longrightarrow \left\{\begin{array}{l}
\frac{\lambda_{A}}{\sqrt{1-2 \alpha}}+\frac{1}{6} \alpha+\frac{1}{6}=p_{A} \\
\frac{\lambda_{A}}{\sqrt{1-2 \beta}}-\frac{1}{6} \beta+\frac{1}{6}=1 \\
\frac{\lambda_{B}}{\sqrt{1-2 \alpha}}-\frac{2}{3} \alpha-\frac{1}{6}=p_{B} \\
\frac{\lambda_{B}}{\sqrt{1-2 \beta}}-\frac{2}{3} \beta-\frac{1}{6}=1
\end{array}\right.\right.
$$

Optimality at $\tilde{a}_{L}$ for firm $A$ and optimality at $\alpha$ for firm $B$ establish the relationship between atom sizes and location of the intervals:

$$
\left\{\begin{array}{l}
p_{A}=1-\frac{\alpha}{2}  \tag{9}\\
p_{B}=\frac{1}{4}+\frac{1}{2} \alpha
\end{array}\right.
$$

All six equations in (8) and (9) pins down all parameters: $\left(\alpha, \beta, \lambda_{A}, \lambda_{B}, p_{A}, p_{B}\right) \approx(0.4607,0.4808,0.1475$, $0.2912,0.7696,0.5652)$. Equilibrium strategies $\left(G_{A}, G_{B}\right)$ are plotted in Figure 4

Figure 4: mixed strategy equilibrium in the example


Proposition 4 shows that any strategy obtained by the algorithm is indeed an equilibrium.

Proposition 4. Any strategy profile that is generated by the algorithm is a mixed strategy equilibrium.

Proof. Proof of case 1 is presented here. Case 2 can be proved in a similar way. Suppose that we obtain $\left(G_{A}, G_{B}\right)$ through equation (3)~(4). By construction, player $A$ gets the same payoff on $\tilde{a}_{L} \cup[\alpha, \beta]$. Player $A$ has no profitable deviation to $[0, \alpha)$ because player $B$ chooses $b>\alpha$ with probability 1 and $\tilde{a}_{L}$ is the best response to $G_{B}$ on $[0, \alpha)$. Player $A$ has no
profitable deviation to $(\beta, 1)$ because player $B$ chooses $b \leq \beta$ with probability 1. Therefore, any design $a>\beta$ is strictly dominated by $\beta$ for player $A$.

For player $B$, first notice that $\tilde{b}_{R} \leq \beta$, which implies player $B$ has no profitable deviations to $(\beta, 1]$ because player $A$ chooses $a<\beta$ with probability 1 and $\tilde{b}_{R}$ is the best response to that. Player $B$ also has no profitable deviations to $[0, \alpha)$. For $b \in\left(\tilde{a}_{R}, \alpha\right)$, because of (5) and strict concavity of expected utility, player $B$ has no profitable deviations to ( $\tilde{a}_{R}, \alpha$ ).

Since both players have no profitable deviations, $\left(G_{A}, G_{B}\right)$ is a Nash equilibrium.

Lastly, proposition 5 talks about uniqueness.

Proposition 5. Every creative contest has a unique regular Nash equilbrium, characterized by Theorem 1 and the proposed algorithm.

## 4 Application - Information Disclosure

As an application of the framework I have developed, I examine the following question. If the contest organizer could strategically choose whether to disclose her ideal design $s^{*}$. Would she want to do so? This section shows that the answer is sometimes "no."

### 4.1 Setting

Consider a symmetric creative contest in which firms have initial designs $\alpha_{0}=0$ and $\beta_{0}=1$ and the same quadratic cost function $c(d)=t d^{2}$.

The contest organizer aims to minimize the distance between the winning design and her ideal design. When the winning design is $s_{w}$, her utility is $u_{o}\left(s_{w} ; s^{*}\right)=-\left|s_{w}-s^{*}\right|$. Prior to learning her ideal design $s *$, the contest designer commits whether to disclose it or not. ${ }^{4}$

[^3]
### 4.2 Concealing $s^{*}$

When $s^{*}$ is concealed, two firms compete in a symmetric creative contest. According to Proposition 2, PSNE always exists. When costs are high $\left(t>\frac{1}{2}\right.$, or $\left.\tilde{a}=\frac{1}{4 t}<\frac{1}{2}\right)$, both firms move $\frac{1}{4 t}$ towards each other. When costs are low $\left(t \leq \frac{1}{2}\right)$, both firms move to exact $\frac{1}{2}$.

### 4.3 Disclosing $s^{*}$

Solving for equilibria when $s^{*}$ is disclosed is harder than it may initially appear because of the interaction between potential head starts and bid caps.

To demonstrate, consider the following scenario: when $s^{*}$ is revealed to be 0.6 , this means firm $B$ is now the advantaged firm because his initial design is closer to $s^{*}$ compared with firm $A$. The difference in their distances to $s^{*}, \Delta=\left|s^{*}-\left(1-s^{*}\right)\right|=0.6-(1-0.6)$, is the size of the head start firm $B$ has. Equilibrium analysis of standard all-pay contests with the sole presence of head starts can be found in Siegel (2014) and Zhu (2019).

To see why bid caps might be in effect, consider any case where costs are close to zero. Both firms choose $s^{*}$ with probability one in equilibrium. Notice that both firms do not gain extra probability of winning by over investing. Therefore, $s^{*}$ is effectively a bid cap for both firms. Equilibrium analysis of standard all-pay contests with the sole presence of bid caps can be found in Che and Gale (1998).

Figure 5 above illustrates the most complicated case when $t$ is moderate. It shows asymmetry level $\Delta=\left|s^{*}-\left(1-s^{*}\right)\right|$ affects equilibrium structure due to the interactions between bid caps and head starts. Detailed equilibrium strategies along with a more general version of the analysis are included in Appendix F. Because of symmetry, let us focus on $s^{*}>\frac{1}{2}$.

- When $\Delta$ is large enough, the head start is large enough such that the weaker player, firm $A$, has no incentive to compete at all. Therefore both firms make no effort in equilibrium. The threshold, $\Delta_{4}$, satisfies $c\left(\Delta_{4}\right)=1$.

Figure 5: 5 cases of asymmetry levels $\Delta$


- As $\Delta$ decreases, there is a level of asymmetry below which the "implicit bidding cap" is hit. This threshold $\Delta_{3}$ is characterized by $c\left(s^{*}\right)=c\left(\frac{1+\Delta_{3}}{2}\right)=1$.
- When $\Delta$ is close enough to zero, both firms choose $s^{*}$ in equilibrium. This happens when $\Delta \leq \Delta_{1}$, which is characterized by $c\left(s^{*}\right)=c\left(\frac{1+\Delta_{1}}{2}\right)=\frac{1}{2}$.
- Unlike Che and Gale (1998), there is another threshold $\Delta_{2}$ between $\Delta_{1}$ and $\Delta_{3}$. Below this threshold, in equilibrium each firm randomizes on two submissions: their featured design and $s^{*}$. When $\Delta>\Delta_{2}$, in equilibrium firms randomize in a similar way to Che and Gale (1998). This threshold $\Delta_{2}$ is determined by equating the size of the head start and the upper bound of the interval on which both firms randomize.

Once we know equilibrium strategies, namely how to firms compete, we can then calculate the distribution of the winning submission and the organizer's utility. Figure 6 below plots organizer's utility for different $s^{*}$ when cost parameter $t$ is low, moderate and high.

In these figures, we can see that when adjustment costs are low, disclosing the ideal design yields the first-best outcome for the contest organizer because no matter where the ideal design is, the winning design will match it. In the middle panel, where adjustment costs are moderate, disclosure of the ideal design is beneficial if it is close to the $\frac{1}{2}$. When the ideal

Figure 6: point-wise policy comparison for different $t$

design is close to one of the firms, disclosure discourages both firms from competing and therefore makes the contest organizer worse off. Comparing between the middle and right panel, we can see that his discouragement effect is more salient when adjustment costs are higher, as disclosure makes the organizer worse off for a wide range of $s^{*}$ in the right panel. Notice that when the organizer makes disclosure decisions, she does not know the realized value of $s^{*}$. Therefore, her ex-ante expected utility will be the average utility over different $s^{*}$, which will be discussed in the next section.

### 4.4 Policy Comparison

In this section, I compare the organizer's utility under no disclosure and full disclosure, and conclude that information disclosure is beneficial to the organizer only when adjustment costs are low.

Proposition 6. There is a threshold $t^{*}$. When $t<t^{*}$, disclosing $s^{*}$ makes the organizer better off by disclosing $s^{*}$. When $t>t^{*}$, disclosing $s^{*}$ makes the organizer worse off.

Figure 7 plots the organizer's expected utility under two policies. The organizer benefits from disclosing $s^{*}$ when adjustment costs are low. There is, however, a countervailing force: once the organizer's ideal design is disclosed, one firm will have an advantage, which discourages the opponent from competing. In consequence, the advantaged firm has weaker incentives to compete. This discouragement effect dominates the benefit of disclosing when adjustment
costs are high. Therefore, when $t>t^{*} \approx 20.54$, disclosing the ideal design makes the organizer worse off.

Figure 7: policy comparison: Disclose vs Conceal with different adjustment costs

| Utility |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.00 | 5 | 10 | 15 | 20 | 25 | 30 |
| -0.05 |  |  |  |  |  |  |
| -0.10 |  |  |  |  |  |  |
|  |  |  |  |  | Disclose |  |
| -0.15 首 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| -0.20 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| -0.25 |  |  |  |  |  |  |
| -0.30 |  |  |  |  |  |  |

## 5 Laboratory Experiment

Thus far, I introduced a model in which contestants were uncertain about the contest organizer's preferences. As an application, I have also investigated whether or not a contest organizer should disclose her preferences and I found that disclosure is not always optimal. Two concerns arise about my model's relevance beyond theory. One concern is that the model assumes rationality and risk neutrality, which are usually not satisfied in the field. Another concern is that the experimental literature has found that subjects tend to overbid in contests and if subjects also overbid in creative contests, specifically, then the model's welfare prediction about information disclosure may no longer hold. ${ }^{5}$

To address these concerns, I conduct a laboratory experiment and ask the following questions: How well does the model predict subjects' behaviors? Do subjects overbid in creative contests? And does the welfare prediction about information disclosure hold in the lab?

[^4]
### 5.1 Experimental Design

I framed the experiment as a location choice problem. Compared with the creative contest framing, this framing is equivalent in theory but allows me to control for subjects' adjustment costs and to quantify the their efforts. Specifically, each subject acted as a shop owner and was paired with another subject. One of them was located at Location 0 of a street of and the other was at Location 100. They decided how much to spend on moving to a new location to attract a consumer. Moving costs were quadratic in moving distance. Specifically, moving to Location $x$ cost $t x^{2}$ to subjects initially located at 0 and $t(100-x)^{2}$ to subjects initially located at 100, where $t$ is a cost parameter. Subjects in a group made simultaneous moving decisions. After that, a programmed consumer arrived at a location and brought a payoff of 100 points to the closest shop owner. Figure 8 shows a decision screen. As subjects moved their mouses, a box updated the corresponding costs in real time.

Figure 8: decision screen

## Round 2 of 3



I implemented a between-subject $2 \times 2$ design, summarized in Table 1. The first treatment variable is subjects' knowledge about consumer's locations. In the Conceal treatments (C,
for short), subjects(i.e. shop owners) were not aware of consumers' locations, but expected that consumers would arrive at given any location with equal probability. In the Disclose treatments (D, for short), the exact locations of the consumers were known to the subjects.

Table 1: experimental design summary

| Treatment Name | Location Disclosed | Moving Cost | \# of Sessions | \# of Subjects |
| :---: | :---: | :---: | :---: | :---: |
| CH | No | High | 2 | 60 |
| CL | No | Low | 2 | 60 |
| DH | Yes | High | 3 | 60 |
| DL | Yes | Low | 3 | 60 |

Another treatment variable is (subject-incurred) moving cost. Translating results in Section 4.4 into the location choice framing, when moving costs are low, the consumer is better off by disclosing her location. When shops' moving costs are high enough, however, disclosing locations makes the consumer worse off, because the shop owner may decide that a move is not worthwhile. However, the threshold value is so high that when used, subjects can hardly afford any move. ${ }^{6}$ Besides, the predicted gain to consumers for concealing information is positive in theory, but almost indistinguishable; it is likely to be overshadowed by the noisy behaviors in the experiment. As a workaround, I chose low- and high- cost parameters to be $1.25 \%$ and $8 \% .^{7}$ In other words, moving a distance of $d$ costs $0.0125 \times d^{2}$ in Low-cost treatments and $0.08 \times d^{2}$ in High-cost treatments (L and H for short). With these parameters, theory predicts that when moving costs are low, the benefit to consumers of disclosing their location is sizable. When moving costs are high, meanwhile, the benefit of disclosing their location is close to zero, as Figure 7 shows.

I conducted a balanced design across all 4 treatments. Each treatment consisted of 2 or 3 independent sessions, adding up to 60 subjects. At the beginning of each session, subjects were randomly assigned into groups of 2 , which remained fixed across all 40 periods. Two of the 40 periods were randomly selected for payment. After subjects finished their location-choice

[^5]tasks, they completed 10 incentivized Holt and Laury (2002) lottery choices, followed by a post-experiment survey about demographics and personality traits. A sample of instructions is included in Appendix H.

Besides holding treatment size constant, I also controlled for the 40 locations where consumers arrived in the following way: first, I randomly drew 40 locations from $U[0,100]$ prior to the start of the first session. Next, I derived 30 random permutations of the 40 locations and then I one-to-one mapped 30 groups in any treatment to these 30 permutations.

In total, I conducted 10 independent computerized sessions (programmed in oTree) at the CBER lab at Wuhan University from March 2019 to May 2019. All 240 subjects were students at Wuhan University. Each session lasted about 70 minutes and the average earning was $¥ 52$ (about $\$ 7.6$ ), including a show-up fee of $¥ 10$ (about $\$ 1.5$ ). ${ }^{8}$

### 5.2 Results

Figure 9: average moving distance by period


[^6]Figures 9 plots the relationship between average moving distance and period in the 4 treatments. In the graph, subjects' moving distances exhibit decreasing trends during the first 20 periods and seems to stabilize at Period 20. Therefore, I will focus the rest of the analyses on Periods 21-40.

Moreover, in all individual-level analyses, I will flip decisions made by subjects who are initially located at 100 . For instance, if a subject with an initial location 100 moves to Location 80 when the consumer at Location 60, the decision will be treated as if the subject is with an initial location 0 and moves to Location 20 (100-80) when the consumer is at Location 40 (100-60).

The analyses are presented as follows. Section 5.2.1 focuses on individual behavior and examines comparative static predictions derived from the theory. Section 5.2.2 is focused on group-level performance and tests the welfare prediction regarding information disclosure. Section 5.2.3 discusses whether subjects overbid in creative contests, and it documents two findings related to overbidding.

### 5.2.1 Moving Distances

This section analyzes subjects' behavior at the individual level. First, I inspect moving decisions in the two Conceal treatments. Figure 10 plots the average moving distance when consumers are ex-post revealed at different locations. The theory predicts that firms move 3.125 units in the CH treatment and 20 units in the CL treatment in equilibrium, these predictions indicated by solid horizontal lines. In other words, subjects move more when moving costs are low. This prediction is supported by experimental data and formalized below.

Result 1: In the conceal treatments, subjects move more when moving costs are low.
Support: Subjects in CH treatment move 6.49 on average, which is significantly lower than 30.89 in CL treatment. (p-value is less than 0.001 , two-sided t-test)

Figure 10: conceal treatments: moving distance vs consumer locations


Notes: Solid lines depict Equilibrium predictions.

Moving to the two Disclose treatments, theoretical predictions about equilibrium behaviors are more complicated. Figure 11 plots the average moving distance when consumers are disclosed at different locations. The solid lines show equilibrium predictions and follow a similar pattern. When the consumer is disclosed to be close to one of the subjects, neither of the subjects has any incentive to move; disclosing the location creates a large advantage for one subject while simultaneously making it clear that it is too costly for the other subject to attempt to catch up. On the other hand, when the consumer is disclosed to be close to the midpoint, the two subjects are at similar distances from the consumer, and, consequently, they compete hard for the prize, randomizing over a range of locations. In this case, the mean of the equilibrium strategy is plotted in the figure. Detailed analysis can be found in Section 4.3 and Appendix F.

The hollow symbols represent the average moving distance of all subjects who share the same distance from a consumer. We can observe that on average, subjects' moving distances respond to consumer locations in a way that is broadly consistent with equilibrium predictions, as formalized in the following result.

Result 2: Subjects move more as the consumer is further away, as long as the distance to the consumer is below a particular threshold. When the distance to the consumer is above

Figure 11: disclose treatments: moving distance vs consumer locations


Notes: Solid lines depict equilibrium predictions.
the threshold, moving distance decreases. ${ }^{9}$
Support: I explain in detail how I analyze the data in the DH treatment.
Let Dist $_{i t}$ denote subject $i$ 's distance from a consumer in period $t$, and Move ${ }_{i t}$ denote subject $i$ 's moving distance in period $t$. Following the theoretical predictions, I construct two dummy variables. $\operatorname{Far}_{i t}$ equals 1 if the consumer is close to one player, namely Dist< 32.32 or Dist $>67.68$. Theory predicts that if $\operatorname{Far}_{i t}=1$, Move $_{i t}=0$. Another dummy is I(Dist $\left.>50\right)$, which captures the different impact of consumer locations on Move ${ }_{i} t$.

I run the following panel regression:
$\operatorname{Move}_{i t}=\alpha+\operatorname{Dist}_{i t}\left(\beta_{0}+\beta_{1} \mathrm{I}(\text { Dist }>50)_{i t}+\beta_{2} \times \operatorname{Far}_{i t}+\beta_{3} \times \operatorname{Far}_{i t} \times \mathrm{I}(\right.$ Dist $\left.>50)\right)+\gamma \operatorname{Control}_{i}+v_{i t}$

Because my focus is on how distances to consumers affect subjects' move decisions, the main parameters of interest are $\beta_{0}$ and $\beta_{1}$. $\beta_{0}$ captures the marginal effect of distance from consumers on moving decisions when distances are below the threshold. $\beta_{0}+\beta_{1}$ captures the marginal effect of distance from consumers on moving decisions when distances are above the threshold. Table 2 summarizes the regression results and shows that $\hat{\beta}_{0}=0.972$ and

[^7]$\hat{\beta}_{1}=-1.724 . H_{0}: \beta_{0}=0$ and $H_{0}: \beta_{0}+\beta_{1}=0$ are both rejected at $5 \%$ significance level (p-values are both smaller than 0.001, two-sided t-test and Wald test, respectively).

### 5.2.2 Consumer Welfare

This section analyzes group level data and examines whether the consumer might benefit from disclosing her location. According to the theoretical predictions in Section 4.4, given the cost parameters in the experiment, the benefit attendant to location disclosure is positive but marginal, and ultimately insignificant, when moving costs are high. The benefit is more substantial (and is significant) when moving costs are low.

Figure 12 plots the mean of consumers' welfare in different treatments, with error bars representing $95 \%$ confidence intervals. In the figure, we can see that in both high cost and low cost scenarios, disclosing location improves consumers' welfare. The improvement is statistically insignificant when moving costs are high, but it is statistically significant when moving costs are low.

Result 3: Consumer's welfare gain is positive but insignificant when moving costs are high. The gain is significant when moving costs are low.

Support: I calculate consumer's average welfare in each group in the last 20 periods, and then compare between treatments.

When moving costs are high, disclosing locations brings the average consumer's welfare from -20.07 (the CH treatment) to -19.28 (the DH treatment). The welfare gain is 0.79 , which is not significant. (p-value is 0.36 , two-sided t-test).

When costs are low, disclosing locations brings the average consumer's welfare from -17.77 (the CL treatment) to -2.30 (the DH treatment). The welfare gain is 15.48 , which is significant. ( p -value is less than 0.001 , two-sided t -test).

Table 2: panel regressions of moving distance on distance to the consumer


Figure 12: consumer's welfare comparison: Conceal vs Disclose


### 5.2.3 Overbidding

The experimental literature has found that, in may instances, lab subjects tend to overbid in contests. In this section, I analyze overbidding behavior in my experiment and highlight two new phenomena that may be worth more attention in the future. I use the term "overbid", here, when a subject in my experiment moves more than the equilibrium predicts. ${ }^{10}$

Table 3 summarizes both the fraction and the scale of overbidding in each treatment. When calculating the fraction, I identify a decision to be overbidding if the moving distance is $20 \%$ more than the equilibrium predicts. ${ }^{11}$

Result 4a: Overbidding is reduced when consumer locations are disclosed.
Support: We can see from Table 3 that subjects overbid in all treatments. The discrepancy

[^8]Table 3: overbidding in different treatments

|  | All | CH | DH | CH vs DH | CL | DL | CL vs DL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Obs. | 240 | 60 | 60 |  | 60 | 60 |  |
| Avg. Size $^{a}$ | 4.03 | 3.36 | 1.32 |  | 10.89 | 0.56 |  |
|  | $p<0.001$ | $p<0.001$ | $p=0.008$ | $p<0.001$ | $p<0.001$ | $p=0.362$ | $p<0.001$ |
| Fraction $^{b}$ | $53.75 \%$ | $61.67 \%$ | $61.67 \%$ |  | $76.67 \%$ | $15.00 \%$ |  |
|  | $p<0.001$ | $p<0.001$ | $p<0.001$ | $p>0.999$ | $p<0.001$ | $p<0.001$ | $p<0.001$ |

a Avg. moving distance above equilibrium. p-values from two-sided t-test.
b Percentage of individuals who overbid. p-values from two-sided propotion test.
between equilibrium prediction and subjects' decisions is significant in all except the DL treatment. When comparing the size of overbidding between the Disclose treatments and the Conceal treatments, I find that when costs are high, disclosing consumer locations reduces overbidding by $3.36-1.32=2.04$, which is statistically significant. (p-value is less than 0.001 ; two-sided t-test). When costs are low, disclosing locations reduces overbidding by $10.89-0.56=10.33$, which is statistically significant. ( p -value is less than 0.001 ; two-sided t-test)

Next, I focus on the two Disclose treatments, because Figure 10 seems to suggest that overbidding behaviors are different when subjects are disclosed to have an advantage vs. when they are disclosed to have a disadvantage. This observation is verified and summarized below.

Result 4b: In two conceal treatments, subjects overbid less when they are disadvantaged.
Support: I divide data in each Disclose treatment into two parts. For example, an observation is in $\mathrm{DH}^{+}$if the consumer is disclosed closer to the subject, namely Dist $_{i t}<50$. Similarly, an observation is in $\mathrm{DH}^{-}$for observations in which the consumer is disclosed closer to the opponent, namely Dist $_{i t}>50$. Notice that a subject could be in $\mathrm{DH}^{+}$in some periods, and in $\mathrm{DH}^{-}$in the other periods.

I then use paired t-test to see whether there is a difference in overbidding when subjects are in different positions. Table 4 summarizes the results.

Table 4: Overbidding in disadvantageous and advantageous positions

|  | $\mathrm{DH}^{+}$ | $\mathrm{DH}^{-}$ | $\mathrm{DH}^{+} \mathrm{vs} \mathrm{DH}^{-}$ | $\mathrm{DL}^{+}$ | $\mathrm{DL}^{-}$ | $\mathrm{DL}^{+} \mathrm{vs} \mathrm{DL}^{-}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Obs. | 60 | 60 |  | 60 | 60 |  |
| Avg. Size | 2.49 | 0.16 |  | 2.51 | -1.33 |  |
|  | $p<0.001$ | $p=0.709$ | $p<0.001^{a}$ | $p<0.001$ | $p=0.174$ | $p<0.001^{a}$ |

$\mathrm{DH}^{+}$and $\mathrm{DL}^{+}$Periods when distances to consumers are less than 50 in respective treatments.
$\mathrm{DH}^{-}$and $\mathrm{DL}^{-}$Periods when distances to consumers are more than 50 in respective treatments.
a These p-values are from two-sided paired t-test.

From the table, we see that in the DH treatment, subjects on average overbid 2.49 when they are advantaged, which is significantly greater than 0.16 when they are disadvantaged. ( p -value is less than 0.001 , two-sided t -test).

In the DL treatment, subjects on average overbid 2.51 when they are advantaged, which is significantly greater than -1.33 when they are disadvantaged. ( p -value is less than 0.001 , two-sided t-test).

### 5.3 Experiment Takeaways

In summary, the experimental results deliver two messages. First, subjects' overbidding behavior is robust. Notice that the experiment departs from the contest literature in the following sense: it is framed in a Hotelling model and it is more challenging to for subjects to learn in this experiment. In the Conceal treatments, subjects are uncertain about where the consumers will be, while in the Disclose treatments, subjects are faced with different disclosed consumer locations. Despite these departures, I find that the average sizes of overbidding are positive in all four treatments, and significant in all but the DL treatment. I also discover two phenomena pertaining to overbidding: disclosing locations reduces overbidding, and being disclosed to be in a disadvantaged positions further reduces overbidding.

The second message in these results is that, although subjects overbid in the experiment, the experimental data are consistent with the model's predictions. This finding offers supporting evidence about the model's empirical relevence.

## 6 Extensions and Discussions

### 6.1 Model Generalization

All equilibrium characterization results go through when $\alpha_{0}$ and $\beta_{0}$ are not restricted.
In the previous analysis, I characterize the equilibrium using only $\tilde{a}_{L}$ and $\tilde{b}_{R}$ from Definition 1. The other two definitions $\tilde{a}_{R}$ and $\tilde{b}_{L}$, are needed when initial designs $\alpha_{0}$ and $\beta_{0}$ are not at endpoints.

Parallel to Theorem 1, PSNE is characterized as follows.
Theorem 1'. The unique PSNE is $\left(\frac{1}{2}, \frac{1}{2}\right)$ if and only if $\tilde{a}_{L} \geq \frac{1}{2} \geq \tilde{b}_{R}$ and $\tilde{a}_{R} \leq \frac{1}{2} \leq \tilde{b}_{L}$. The unique PSNE is $\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$ if and only if $\tilde{a}_{L}<\tilde{b}_{R}, U_{A}\left(\tilde{a}_{L}, \tilde{b}_{R}\right) \geq \lim _{a \downarrow \tilde{b}_{R}} U_{A}\left(a, \tilde{b}_{R}\right)$ and $U_{B}\left(\tilde{a}_{L}, \tilde{b}_{R}\right) \geq \lim _{b \uparrow \tilde{a}_{L}} U_{B}\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$. In other cases, there exists no PSNE.

The only difference from Theorem 1 is the addition of $\tilde{a}_{R} \leq \frac{1}{2} \leq \tilde{b}_{L}$ as a condition when $\left(\frac{1}{2}, \frac{1}{2}\right)$ is a PSNE. It is always satisfied when $\alpha_{0}=0$ and $\beta_{0}=1$ because $\tilde{a}_{R}=0$ and $\tilde{b}_{L}=1$ in this case.

To see why $\tilde{a}_{R} \leq \frac{1}{2} \leq \tilde{b}_{L}$ is true in an PSNE, suppose $\tilde{a}_{R}>\frac{1}{2}$. $\left(\frac{1}{2}, \frac{1}{2}\right)$ can not be an equilibrium because firm $A$ is better off by deviating from $\frac{1}{2}$ to $\tilde{a}_{R}$. Similarly, if $\tilde{b}_{L}<\frac{1}{2}$, firm $B$ is better off by deviating from $\frac{1}{2}$ to $\tilde{b}_{L}$.

Parallel to Proposition 3, support of MSNE is characterized as follows.
Proposition 3': In equilibrium, both firms continuously randomize on a common interval $[\alpha, \beta]$. Moreover, each firm has at most one atom:

1. If $\tilde{a}_{L}>\tilde{b}_{R}$ and $\tilde{a}_{R}>\frac{1}{2}$, firm $A$ 's atom is $\tilde{a}_{R}$ and firm $B$ 's atom is $\beta$, satisfying $\tilde{a}_{R}>\beta>\alpha>\frac{1}{2}$.
2. If $\tilde{a}_{L}>\tilde{b}_{R}$ and $\tilde{b}_{L}<\frac{1}{2}$, firm A's atom is $\alpha$ and firm $B$ 's atom is $\tilde{b}_{L}$, satisfying $\tilde{b}_{L}<\alpha<$ $\beta<\frac{1}{2}$.
3. Otherwise, the same as in Proposition 3.

Again, when $\alpha_{0}=0$ and $\beta_{0}=1$, cases 1 and 2 in Proposition 3' can not occur.
Lastly, Assumption 1 can be relaxed to the following assumption.
Assumption 1': $c_{i}$ is increasing, twice differentiable, and strictly convex.
All results, except for the direct proof of existence of regular MSNE in Appendix E, extend with Assumption 1'. It is challenging to prove the existence of regular MSNE when Assumption $1^{\prime}$ is satisfied, but I conjecture it to be true because all existing all-pay contest models satisfying Assumption 1' have regular MSNE.

### 6.2 Applications in Other Fields

In this paper, I have demonstrated two applications of the model in contest settings. This model can be applied in many other fields as well. One such field is industrial organization, in which the model is a generalization of a Hotelling location model. It can be used to understand interactions between upstream and downstream firms in a supply chain. For example, an upstream firm wishes to procure products from one of two potential suppliers. The upstream firm can either be specific or vague about its needs, and the two suppliers will create prototypes, then, based on which one of the suppliers will be picked. With appropriate empirical data, my model can help quantify the impact of information disclosure in supply chains.

Another topic my model speaks to is electoral voting, as in Wittman (1983) and Calvert (1985). Two candidates participate in an election by each choosing a policy as a platform. According to the median voter theorem, under a majority-rule voting system, the winner is selected by the median voter. Because the adjustment costs in my model always depend on firms' own submissions, my model does not precisely combine both office and policy motivations. However, instead, it can be interpreted as a voting model with office-motivated
candidates, who need to pay a cost to change platforms (e.g. advertising or campaigning costs).

## 7 Conclusion

This paper introduces a simple model of creative contests, in which participants are uncertain about the organizer's preferences. I characterize the unique Nash equilibrium. As an application, I consider whether the contest organizer should disclose her ideal design, and I find that disclosing is not always optimal, as it creates asymmetry between firms and hence discourages competition. Based on this model, I also conduct a laboratory experiment and find that, although subjects overbid, experiment results are broadly consistent with comparative static predictions and with the welfare prediction about information disclosure.

This study points toward several directions for future research. One direction is to consider a contest organizer's optimal disclosure policy either in the Bayesian persuasion framework [Kamenica and Gentzkow (2011)] or in the disclosure game framework [Grossman (1981) and Milgrom (1981)]. Another direction is to utilize the model in laboratory experiments involving creativity, as in Charness and Grieco (2018) and references therein. Lastly, as discussed in Section 6.2, the model can be applied to other fields. For example, it speaks to an electoral voting setting, wherein office-motivated candidates faces costs for changing their platforms, or to an industrial organization setting, wherein an upstream firm decides whether it should specify its needs when procuring products from multiple downstream firms.

## Appendices

## A Proof of proposition 1

First, I include Theorem 5 of Dasgupta and Maskin (1986) as Lemma 5 below.
Lemma 5. Let $A_{i} \subset R^{1}(i=1, \ldots, N)$ be a closed interval and let $U_{i}: A \rightarrow R^{1}(i=$ $1, \ldots, N)$ be continuous except on a subset $A^{* *}(i)$ of $A^{*}(i)$, where $A^{*}(i)$ is explained below. Suppose $\sum_{i=1}^{N} U_{i}(a)$ is upper semi-continuous and $U_{i}\left(a_{i}, a_{-i}\right)$ is bounded and weakly lower semi-continuous in $a_{i}$. Then the game $\left[\left(A_{i}, U_{i}\right) ; i=1, \ldots, N\right]$ possesses a mixed-strategy equilibrium.

Definitions in Lemma 5:

1. $A^{*}(i)$ is continuous manifold of dimension less than $N$ containing discontinuous points $A^{* *}(i)$.
2. $U_{i}\left(a_{i}, a_{-i}\right)$ is weakly lower semi-continuous in $a_{i}$ if $\forall \bar{a}_{i} \in A_{i}^{* *}(i), \exists \lambda \in[0,1]$ such that $\forall a_{-i} \in A_{-i}^{* *}\left(\bar{a}_{i}\right), \lambda \lim \inf _{a_{i} \uparrow \bar{a}_{i}} U_{i}\left(a_{i}, a_{-i}\right)+(1-\lambda) \lim \inf _{a_{i \downarrow} \downarrow \bar{a}_{i}} U_{i}\left(a_{i}, a_{-i}\right) \geq U_{i}\left(\bar{a}_{i}, a_{-i}\right)$

To apply Lemma 5, I verified that all conditions are met in creative contests. $A_{i}=[0,1]$, $A^{*}(i)=\{(x, x): x \in[0,1]\}$ and $A^{* *}(i)=A^{*}(i) \backslash\left\{\left(\frac{1}{2}, \frac{1}{2}\right)\right\} . \sum U_{i}(a)$ is continuous because it is a fixed prize minus sum of costs. $U_{i}$ is bounded because both the prize and costs are bounded. To see $U_{i}(a)$ is weakly lower semi-continuous, note that $A_{-i}^{* *}\left(\bar{a}_{i}\right)=\left\{\bar{a}_{i}\right\}$, and moreover $U_{i}\left(\bar{a}_{i}, \bar{a}_{i}\right)$ always lies between $\lim _{a_{i} \uparrow \bar{a}_{i}} U_{i}\left(a_{i}, \bar{a}_{i}\right)$ and $\lim _{a_{i} \downarrow \bar{a}_{i}} U_{i}\left(a_{i}, \bar{a}_{i}\right)$. There exists $\lambda$, either 0 or 1 , such that inequality in the definition of weakly lower semi-continuous holds. Because all conditions in Lemma 5 are all satisfied, every creative contest has a Nash equilibrium.

## B Proof of Claim 1

First I prove there are only two candidate PSNE.

Lemma 6. Every creative contest has only two candidate PSNE: $\left(\frac{1}{2}, \frac{1}{2}\right)$ and $\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$.

Proof. I first prove a weaker result. That is, in equilibrium, players strategy must weakly reserve the order of their initial designs.

Claim 3. Every pure strategy Nash equilibrium $\left(a^{*}, b^{*}\right)$ in creative contests satisfies $a^{*} \leq b^{*}$.

Proof. Suppose otherwise there exists a pure strategy Nash equilibrium with $a^{*}>b^{*}$. If $a^{*}>\alpha_{0}$, then firm $A$ is better off reducing his adjustment. This will increase his chance of winning, reduce his bidding cost and therefore yield a higher expected utility. On the other hand, if $a^{*} \leq \alpha_{0}$, this implies $b^{*}<a^{*} \leq \alpha_{0}<\beta_{0}$. By similar reasoning, firm $B$ is better off reducing his adjustment. Therefore, $a^{*}>b^{*}$ always leads to contradictions.

This completes the proof that in a pure strategy Nash equilibrium, it must be $a^{*} \leq b^{*}$.

Now I examine the case of $a^{*}=b^{*}$ and $a^{*}<b^{*}$ separately. When $a^{*}=b^{*}$ is an equilibrium, they must both equal $\frac{1}{2}$, otherwise both firms strictly prefer one side to another, while cost difference is negligible by continuity.

When $a^{*}<b^{*}$ is an equilibrium, it must be $a^{*} \geq \alpha_{0}$ because otherwise $a^{*}$ is strictly dominated by $\alpha_{0}$ because the latter increases probability of winning while decreases adjustment costs. Similar logic implies $b^{*} \leq \beta_{0}$. By definition of $\tilde{a}_{L}$ and $\tilde{b}_{R}$, it must be $a^{*}=\tilde{a}_{L}$ and $b^{*}=\tilde{b}_{R}$.

By Lemma 6 and Claim 3, when $\tilde{a}_{L} \geq \tilde{b}_{R}$, the only PSNE candidate is $\left(\frac{1}{2}, \frac{1}{2}\right)$. When $\tilde{a}_{L}<\tilde{b}_{R}$ either $\tilde{a}_{L}<\frac{1}{2}$ or $\frac{1}{2}<\tilde{b}_{R}$, or both are true. Either firm $A$ has a profitable deviation to $\tilde{a}_{L}$ or firm $B$ has a profitable deviation to $\tilde{b}_{R}$, or both. Consequently, $\left(\frac{1}{2}, \frac{1}{2}\right)$ is not an equilibrium.

## C Proof of Proposition 3

I formally prove Lemma 2 to Lemma 4 below.

Let $S P_{i}, C I_{i}$, and $I P_{i}$ denote player $i$ 's support, unions of closed intervals and unions of closed isolated points.

Proof of Lemma 2: To show in equilibrium, both firms randomize on one common intervalin the following steps:

Step 1. Sets of Intervals are nonempty and coincide with each other. That is, $C I_{1}=C I_{2} \neq$ $\emptyset$.

First I prove that they cannot both be empty. Suppose it is true, it implies that both firms' best response sets only consist of isolated points. Because there is no equilibrium in pure strategy, at least one of them, say $I P_{1}$, contains more than two points. Suppose $I P_{1}=\left\{s_{1}^{(1)}, \ldots, s_{n_{1}}^{(1)}\right\}$ where $s_{k}^{(1)}$ is strictly increasing in $k$. Then $I P_{2}$ must have the following property: there exists one and only one element in $I P_{2}$ between every two consecutive elements in $I P_{1}$. Moreover, in $I P_{2}$, there is no point that is less than $s_{1}^{(1)}$ and there is exactly one point that is greater than $s_{n_{1}}^{(1)}$. These two properties come from Lemma 1, increasing cost functions and the fact that $\alpha_{0}<\beta_{0}$. This leads to a contradiction because of $s_{1}^{(1)}<s_{1}^{(2)}<s_{2}^{(1)}$ : $s_{1}^{(1)}<s_{1}^{(2)}$ implies $\alpha_{0}<s_{1}^{(1)}$ and $s_{1}^{(1)}$ being optimal means the marginal gain and marginal costs are equal at this point. At $s_{2}^{(1)}$, however, the marginal gain is less than that at $s_{1}^{(1)}$ while the marginal costs are greater. Hence marginal gain is strictly less than marginal cost at $s_{2}^{(1)}$. In other words, firm $A$ has incentive to move to the left and therefore $s_{2}^{(1)}$ cannot be optimal.

Next I show that given that at least one of them is non-empty, $I P_{1}$ and $I P_{2}$ must coincide. Suppose otherwise, there exists an interval $(l, r)$ belonging to $C I_{1}$ but not to $C I_{2}$. Because firm 2's support contains finite isolated points, it is without loss to assume ( $l, r$ ) does not contain any of them. $(l, r)$ then violates Lemma 1.

This completes the proof that in a mixed strategy equilibrium, firms must randomize on some common intervals.

Step 2: $\frac{1}{2}$ is not in any firm's support. That is, $\frac{1}{2} \notin S P_{i}$.
Suppose otherwise, without loss assume $\frac{1}{2} \in S P_{1}$, which implies either $\frac{1}{2} \in I P_{1}$ or $\frac{1}{2} \in C I_{1}$. First I show $\frac{1}{2} \notin I P_{1}$. Suppose otherwise, by the previous step, there exists some interval
in the support. Without loss assume that $\left[l_{1}, r_{1}\right]$ is the rightmost interval on the left of $\frac{1}{2}$. $r_{1}<\frac{1}{2}$ implies that both firms cannot have atoms at $r_{1}$. Consider the smallest element in $I P_{1} \cup I P_{2} \cap\left(r_{1}, \frac{1}{2}\right]$ and denoted it as $s_{1}$. It's not empty because $\frac{1}{2}$ belongs to the set. Then $r_{1}$ and $s_{1}$ violates Lemma 1, contradiction.

Next I prove that $\frac{1}{2} \notin C I_{1}, \frac{1}{2}$ is special for the following reason: consider firm1, moving from $s$ to its immediate right has two source of marginal gain - marginal gain from local mass and marginal gain from global mass. Marginal gain from global mass is capped by $\frac{1}{2}$ and decreases in $s$. Marginal gain from local mass is equal to the density of its opponent and the location of $s$. When $s=\frac{1}{2}$, this marginal gain is always zero. Let's look at two cases: (1) when $\frac{1}{2}$ is an interior point of the support and (2) when $\frac{1}{2}$ is the boundary point. First suppose that both firms get their equilibrium payoff in an open neighborhood of $\frac{1}{2}$. Due to continuity of cost, for the two firms, their probabilities of choosing $s<\frac{1}{2}$ and $s>\frac{1}{2}$ must equal. This in turn makes equal the marginal gain of moving to both directions starting from $\frac{1}{2}$. That is, marginal gain of moving from $\frac{1}{2}$ to $\frac{1}{2}-\epsilon$ and $\frac{1}{2}+\epsilon$ is symmetric. However, marginal cost cannot be symmetric due to convexity of cost functions, unless $\alpha_{0}=\frac{1}{2}$. The same logic applies to firm2 as well since $C I_{1}=C I_{2}$ from the first step. As a result, it's not possible that both firms randomize continuously around $\frac{1}{2}$. Next to see that $\frac{1}{2}$ cannot the boundary of some interval either, without loss assume there exists a increasing sequence $\left\{l_{k}\right\}$ converging to $\frac{1}{2}$ such that firm1 gets equilibrium payoff at each points, while firm1 does not get equilibrium payoff on $\left(\frac{1}{2}, \frac{1}{2}+\epsilon\right)$ for some small $\epsilon$. This implies that $\alpha_{0}<\frac{1}{2}$. Now consider $\left\{l_{k}\right\} \cap\left(\alpha_{0}, \frac{1}{2}\right)$. Notice that as $l_{k}$ move closer to $\frac{1}{2}$, global marginal gain decreases because firm2 puts more mass on $s<l_{k}$ as $l_{k}$ increases. Local marginal gain also decreases because at $\frac{1}{2}$ the marginal gain is zero. However, marginal costs are increasing. This contradicts the assumption that all $l_{k}$ 's are optimal for firm 1.

This completes the proof that $\frac{1}{2}$ cannot be in any firm's support.
Step 3. There is only one interval in the support. That is, $C I_{1}=C I_{2}=[\alpha, \beta]$.

Suppose that $C I_{i}$ contains more than one interval. In other words, there are gaps in the support. I first show there cannot be any isolated points in these gaps. After that I will prove these intervals must be connected.

Suppose $C I_{i}=\cup_{k=1}^{n}\left[a_{k}, b_{k}\right]$, where $n>1$ and $a_{k+1}>b_{k}$. Consider any gap $\left(b_{k}, a_{k+1}\right)$. For notation simplicity, let $l=b_{k}$ and $r=a_{k+1}$. First there can't be atoms at both $l$ and $r$ because an atom at $l$ requires $l>\frac{1}{2}$ while atom at $r$ requires $r<\frac{1}{2}$. These two cannot hold simultaneously because $l<r$. Moreover, at most one firm could have atom every point since we have proven that $\frac{1}{2}$ is not in the support. Combining these two, we can without loss assume neither of these firms has atom at $r$ and firm2 doesn't have an atom at $l$. This implies firm1 gets his equilibrium payoff at both $l$ and $r$. For both $l$ and $r$ to be optimal, there must exist some points from $I P_{2}$ in $(l, r)$, otherwise $l$ and $r$ violates lemma 1. Consider the largest among these isolated points and denote it by $s^{*} \in I P_{2}$. If there is no point in $I P_{1} \cap\left(s^{*}, r\right)$, then $s^{*}$ and $r$ for firm2 violates lemma 1. If there is, then that point (if multiple, choose any of them) and $r$ for firm1 violates lemma 1.

This completes the proof that both firms randomize on exactly one common interval $[\alpha, \beta]$.
More over, I prove that firms randomize continuously on $[\alpha, \beta]$, that is, there is no interior mass point. Suppose $C I_{i}$ includes a mass point $s_{\text {atom }} \neq \frac{1}{2}$. This creates a jump in the expected utility of the other firm $j$. As a consequence, firm $j$ cannot get her equilibrium payoff on both side of the mass point, contradicting with the previous property that $[\alpha, \beta] \in C I_{j}$.

Proof of Lemma 3:
Loosely speaking, the proof is similar to the proof that support cannot consist of only isolated points in the Step 1 proof of Lemma 2. The trick is that the continuous randomization on interval $[\alpha, \beta]$ can be regarded as one equivalent mass point at its conditional expectation.

A more formal proof is the following. I first show that $\left|I P_{i}\right| \leq 2$, one on each side of $[\alpha, \beta]$. After that I will show isolated points on both sides cannot emerge simultaneously for each player, hence $I P_{i}$ consists at most one point.

Firstly, each firm has at most one isolated point in its support on each side of $[\alpha, \beta]$, implying $\left|I P_{i}\right| \leq 2$. Suppose otherwise that firm1 has two isolated points $s_{1}^{(1)}<s_{2}^{(1)}<\alpha$. To prevent violation of lemma 1 , there is one and only one isolated point for firm2, $s^{(2)}$, such that $s^{(2)} \in\left(s_{1}^{(1)}, s_{2}^{(1)}\right)$. Since the marginal gain at $s_{2}^{(1)}$ is less than at $s_{1}^{(1)}$ while the marginal costs are higher, $s_{2}^{(1)}$ cannot be optimal for firm1. This shows that $I P_{i}$ consists of at most two points, one on each side of $[\alpha, \beta]$,

They cannot emerge simultaneously on both sides: if so, call those two isolated point $s_{1}^{(1)}$ and $s_{2}^{(1)}$. Without loss assume that $\beta<\frac{1}{2}$. As previously shown, there is no atom at $\beta$. To prevent violation of lemma 1 , there is one and only one isolated point between $\beta$ and $s_{2}^{(1)}$. This means firm2 chooses location $s \leq s_{2}^{(1)}$ for sure. $s_{2}^{(1)}$ being optimal implies that $\alpha_{0}>s_{2}^{(1)}$. This leads to contradiction because marginal gain at $s_{2}^{(1)}$ is greater than that of $s_{1}^{(1)}$ while the marginal costs are lower. This completes the proof that there is at most one isolated point for each player.

In fact, at most one firm can have non-empty isolated points. This is because when firm $i$ is active at an isolated point $s_{i}<\alpha$, firm $j$ must have an atom at $\alpha$, otherwise firm $i$ get equilibrium payoff at both $s_{i}$ and $\alpha$, contradicting lemma 1 . An atom at $\alpha$ in turn implies that $\alpha<\frac{1}{2}$ because firm $i$ is better off at $\alpha+\epsilon$ compared with $\alpha-\epsilon$. If firm $j$ is also active at some other isolated point $s_{j}$, it's obvious that $s_{j}>\beta>\alpha$. With the same logic, we can conclude that firm $i$ has an atom at $\beta$ and therefore $\beta>\frac{1}{2}$. This contradicts the result in the second step that $\frac{1}{2}$ is not in the support.

Proof of Lemma 4: here I prove the case where $\tilde{a}_{L}$ and $\tilde{b}_{R}$ are both smaller than $\frac{1}{2}$. The other case can be proved similarly.

First notice that if $\beta<\frac{1}{2}$, there is no atom at $\beta$ because this will make $\beta+$ strictly better than $\beta$ - and contradicts with $[\alpha, \beta]$ in the support. Similarly, if $\alpha>\frac{1}{2}$, there is no atom at $\alpha$.

Next, if player $i$ has an atom at $s<\alpha$, player $j$ must have an atom at $\alpha$. This is because

Lemma 1 implies that player $j$ chooses $(s, \alpha]$ with a positive probability. Moreover, if player $j^{\prime}$ 's item is not $\alpha$ (say $s^{\prime}<\alpha$ ), then player $j$ best responds at both $s^{\prime}$ and $\alpha$, while player $i$ chooses $\left[s^{\prime}, \alpha\right]$ with zero probability, contradicting with Lemma 1. Lastly, it must be one of the two cases: firm $A$ has an atom at $\tilde{a}_{L}$ and player $B$ has an atom at $\alpha$, or firm $A$ has an atom at $\beta$ and player $B$ has an atom at $\tilde{b}_{R}$.

Recall that first order condition for firm $A$ on $[\alpha, \beta]$ is

$$
U_{A}^{\prime}(z)=g_{B}(z)(1-2 z)+\frac{1}{2}-G_{B}(z)-c_{A}^{\prime}(z)-p_{B}
$$

$\tilde{a}_{L}<\frac{1}{2}$ implies that $U_{A}^{\prime}\left(\frac{1}{2}+\right)<0$. This means the interval $[\alpha, \beta]$ must be to the left of $\frac{1}{2}$, firm $A$ has an atom at $\tilde{a}_{L}$ and player $B$ has an atom at $\alpha$.
$\beta>\tilde{b}_{R}$ because otherwise firm $B$ would deviate from $\beta$ to $\tilde{b}_{R}$.

## D Proof of Proposition 5

First, I prove that there is no MSNE when there is PSNE. When the PSNE is $\left(\frac{1}{2}, \frac{1}{2}\right)$, namely $\tilde{a}_{L} \geq \frac{1}{2} \tilde{b}_{R}$. As I have shown in the proof of Lemma 4, the firms' atoms must be in one of the following two cases: firm $A$ has an atom at $\tilde{a}_{L}$ and player $B$ has an atom at $\alpha$, or firm $A$ has an atom at $\beta$ and player $B$ has an atom at $\tilde{b}_{R}$. Consider the first case. First order condition for firm $B$ on $[\alpha, \beta]$ requires that:

$$
U_{B}^{\prime}(z)=g_{A}(z)(1-2 z)+\frac{1}{2}-G_{A}(z)+c_{B}^{\prime}(1-z)-p_{A}
$$

Notice that $\beta>\alpha>\frac{1}{2}$. Consider $z \uparrow \beta$. The above formula cannot be zero because $\lim _{z \uparrow \beta} G_{A}(z)+p_{A}=1$, while $\frac{1}{2}+c_{B}^{\prime}(\beta)<1$. Therefore there is no MSNE.

Now consider PSNE being $\left(\tilde{a}_{L}, \tilde{b}_{R}\right)$. If $\tilde{a}_{L}<\frac{1}{2}<\tilde{b}_{R}$, Lemma 4 leads to contradictions. If firm $A$ has an atom at $\tilde{a}_{L}$, it implies $\beta<\frac{1}{2}$. But it is impossible to meet the requirement that $\beta_{R}<\beta<\frac{1}{2}$. Similarly, it cannot be the case that firm $B$ has an atom at $\tilde{b}_{R}$. If $\tilde{a}_{L}<\tilde{b}_{R}<\frac{1}{2}$,
it must be $\tilde{a}_{L}<\tilde{b}_{R}<\alpha$. Otherwise, consider $U_{B}^{\prime}\left(\tilde{b}_{R}\right)$, it is strictly positive. $\tilde{b}_{R}<\alpha$ also leads to contradiction because $U_{A}\left(\tilde{a}_{L}, \tilde{b}_{R}\right) \geq U_{A}\left(\tilde{b}_{R}+, \tilde{b}_{R}\right)>U_{A}\left(\alpha+, G_{B}\right)$, contradicting equation (4).

Next I show that when there is a MSNE, it is the unique one. Consider the type of MSNE where firm $A$ has an atom at $\tilde{a}_{L}$ and firm $B$ has an atom at $\alpha$. Equilibrium is pinned down by atom sizes, $p_{A}$ and $p_{B}$, and the interval $[\alpha, \beta]$. By equation (3) $\sim(5)$, each $p_{A}$ corresponds to at most one $\left(p_{B}, \alpha, \beta\right)$. Now suppose that there are two different MSNE, namely $p_{A}$ and $p_{A}^{\prime}$ such that equation (3) $\sim(5)$ holds. Without loss assume $p_{A}<p_{A}^{\prime}$. Because firm $A$ chooses with a higher probability, by equation (5), $\alpha^{\prime}<\alpha$. Since firm $A$ is indifferent between $\tilde{a}_{L}$ and $\alpha+$, this means $p_{B}^{\prime}<p_{B}$. By the first order conditions, $\alpha^{\prime}<\alpha$ implies that $G_{A}^{\prime}>G_{A}$ and $G_{B}^{\prime}>G_{B}$. This leads to contradiction because it is not possible for $p_{A}^{\prime}+G_{A}^{\prime}\left(\beta^{\prime}\right)=1$ and $p_{B}^{\prime}+G_{B}^{\prime}\left(\beta^{\prime}\right)=1$ to hold at the same time.

## E Proof of existence of regular MSNE

This section presents a direct proof of existence and uniqueness when costs are quadratic. I will prove the following statement:

Claim 4. If $\tilde{a}_{L}$ and $\tilde{b}_{R}$ are both smaller than $\frac{1}{2}$ and there is no PSNE, equation systems (3) $\sim$ (5) have a unique admissible solution. Namely, $\tilde{a}_{L} \leq \alpha<\beta<\frac{1}{2}, \tilde{b}_{R}<\beta$ and $0 \leq p_{A}, p_{B} \leq 1$. If $\tilde{a}_{L}$ and $\tilde{b}_{R}$ are both greater than $\frac{1}{2}$ and there is no PSNE, equation systems (3) $\sim(5)$ have a unique admissible solution. Namely, $\frac{1}{2} \leq \alpha<\beta<\tilde{b}_{R}, \tilde{a}_{L}>\alpha, 0 \leq p_{A}, p_{B} \leq 1$.

I will illustrate the method with the first case. Within the first case, there are two sub-cases: either $\tilde{a}_{L}>\tilde{b}_{R}$ or $\tilde{a}_{L}<\tilde{b}_{R}$ and $U_{A}\left(\tilde{a}_{L}, \tilde{b}_{R}\right)<U_{A}\left(\tilde{b}_{R}+, \tilde{b}_{R}\right)$. I will illustrate the method with the first sub-case. All rest cases can be proved similarly.

Following the proposed algorithm, Claim 4 can be translated into the following mathematical statement:

Claim 5. Note that $\tilde{a}_{L}=\frac{1}{4 t_{A}}$ and $\tilde{b}_{R}=1-\frac{1}{4 t_{B}}$ When $1-\frac{1}{4 t_{B}}<\frac{1}{4 t_{A}}$ and $t_{A}>\frac{1}{2}$, the following equation system

$$
\left\{\begin{array}{l}
\sqrt{1-2 a}\left(p_{A}-\frac{1}{6}\left(3+4 t_{B}(a-2)\right)\right)=\sqrt{1-2 b}\left(1-\frac{1}{6}\left(3+4 t_{B}(b-2)\right)\right)  \tag{10}\\
\sqrt{1-2 a}\left(p_{B}-\frac{1}{6}\left(3-4 t_{A}(a+1)\right)\right)=\sqrt{1-2 b}\left(1-\frac{1}{6}\left(3-4 t_{A}(b+1)\right)\right) \\
p_{B}=\frac{\left(1-4 a t_{A}\right)^{2}}{16(1-2 a) t_{A}} \\
p_{A}=\frac{1}{2}+2 t_{B}(1-a)
\end{array}\right.
$$

has a unique solution $(a, b)$ such that $\tilde{a}_{L} \leq a<b<\frac{1}{2}, \tilde{b}_{R}<\beta$

Proof. Root existence of polynomial equation systems is a widely studied area in computational algebraic geometry, see Sturmfels (2002).

I prove Claim 5 in two steps: I first show that there exists a unique admissible solution for some particular parameter $\left(t_{A}, t_{B}\right)$. After that I show that the number of real solutions does not change for all parameters $\left(t_{A}, t_{B}\right)$ in the stated area.

The first step is done because example 1 is a special case, with $t_{A}=1$ and $t_{B}=\frac{1}{4}$, and there is unique admissible solution.

To show that number of real roots is always one, I first use the resultant and reduce the system (10) into a univariate case (with two parameters). ${ }^{12}$

Specifically,

$$
\begin{aligned}
G_{0}\left(a ; t_{A}, t_{B}\right)= & t_{A}-120 t_{A}^{2}+216 a t_{A}^{2}+768 t_{A}^{3}-2304 a t_{A}^{3} \\
& +\ldots \\
& -1179648 a^{5} t_{A} t_{B}^{6}+262144 a^{6} t_{A} t_{B}^{6}
\end{aligned}
$$

[^9]The key idea of constant number of real roots comes from continuity: as parameters vary continuously, the solution to the algebraic system varies continuously if treated over the field of complex numbers. The real solution for the system might disappear only if it collides with the other solution. The new real solution might appear only when a pair of complex solutions collide. That means that the necessary condition for bifurcation value for the parameter under which the system might change the number of its real solutions is that this value corresponds to the polynomial $G_{0}$ possessing a multiple root. The condition for the existence of a multiple root for the univariate polynomial is the vanishing of its discriminant.

To help understanding this argument, let's look at a familiar quadratic example: $x^{2}+b x+c=$ 0 , in which discriminant is $\Delta=b^{2}-4 c$. On the complex number fields, number of roots is constant (two), regardless of parameter $b$ and $c$. We know that number of real roots decreases from two to zero as discriminant cross zero from above. The key insight is that as long as discriminant does not vanish, the number of real root will be constant.

The discriminant of $G_{0}\left(a ; t_{A}, t_{B}\right)$ is

$$
\begin{aligned}
D_{a}\left(t_{A}, t_{B}\right)= & -3112 t_{A}^{12}+28984 t_{A}^{13}+353000 t_{A}^{14}+ \\
& +\ldots \\
& -965541888 t_{A}^{7} t_{B}^{17}-1363673088 t_{A}^{8}{ }_{B}^{17}-575668224 t_{A}^{9} t_{B}^{17}
\end{aligned}
$$

For all $\left(t_{A}, t_{B}\right)$ in the interested parameter region, it is negative. Therefore there exists a unique real solution $(a, b)$ to equation system (10).

Lastly, I show the root obtained satisfies constraints (1) $\tilde{a}_{L}<a<b<\frac{1}{2}$ and (2) $\tilde{b}_{R}<b$. The main idea is again continuity. I'll prove the solution satisfy $a<b$, the other constraints can be proved similarly.

Note that in the special case solution when $t_{A}=1, t_{B}=\frac{1}{4}$, the solution of the system satisfies $a<b$. If there is some $\left(t_{A}^{\prime}, t_{B}^{\prime}\right)$ such that the solution of the system violates $a<b$,
by continuity there must exists some $\left(t_{A}^{*}, t_{B}^{*}\right)$ such that the solution of the system satisfies $a=b$. In other words, consider an additional constraint $G_{1}\left(a, b ; t_{A}, t_{B}\right)=a-b$, and equation system (10) have a common root. Repeating the above process, we can show that the system does not have a common root, namely it's not possible to have a common root when $\left(t_{A}, t_{B}\right)$ are in the interested area. This proves that solution to (10) always satisfies $a<b$.

## F Nash Equilibrium When the Ideal Design is Disclosed

I start with the most complicated case, namely moderate cost, as illustrated in Figure 5. In the end, I will define precisely what is low cost, moderate cost and high cost.

Define asymmetry level $\Delta=\left|s^{*}-\left(1-s^{*}\right)\right|$ and I focus on $s^{*}>\frac{1}{2}\left(s^{*}<\frac{1}{2}\right.$ can be solved symmetrically). Moreover, I write equilibrium strategy in terms of effort $e_{i}$. Namely, for submissions $\left(s_{A}, s_{B}\right), e_{A}=s_{A}$ and $e_{B}=1-s_{B}$.

- When $\Delta>\Delta_{4}=c^{-1}(1)=\sqrt{\frac{1}{t}}$, both firms make no effort in equilibrium. $e_{A}=e_{B}=0$
- When $\Delta \in\left[\Delta_{3}, \Delta_{4}\right]$, where $\Delta_{3}=2 c^{-1}(1)-1=2 \sqrt{\frac{1}{t}}-1$, firm $B$ has a head start of size $\Delta$ and the bid cap is not hit. In this case,

$$
G_{A}\left(e ; s^{*}\right)= \begin{cases}w_{B} & \text { if } e<\Delta \\ w_{B}+c(e-\Delta) & \text { if } \Delta \leq e<c^{-1}(1) \\ 1 & \text { if } c^{-1}(1) \leq 1\end{cases}
$$

and

$$
G_{B}\left(e ; s^{*}\right)= \begin{cases}c(e+\Delta) & \text { if } e<c^{-1}(1)-\Delta \\ 1 & \text { otherwise }\end{cases}
$$

, where $w_{B}=1-c\left(c^{-1}(1)-\Delta\right)$ is firm $B$ 's equilibrium payoff.

- When $\Delta \in\left[\Delta_{2}, \Delta_{3}\right]$, this is the case generalized from Che and Gale (1998), where $\Delta_{2}=1-\sqrt{2(t-1) / t}$ solves $\bar{b}=\Delta$, where $\bar{b}$ is the upper bound of randomizing interval and defined below. In this case, firm $A$ have atoms $\Delta$ and $s^{*}$, and firm $B$ have atoms at 0 and $s^{*}-\Delta$ :

$$
G_{A}\left(e ; s^{*}\right)= \begin{cases}0 & \text { if } e<\Delta \\ w_{B}+c(e-\Delta) & \text { if } \Delta \leq e<\bar{b} \\ w_{B}+c(\bar{b}-\Delta) & \text { if } \bar{b} \leq e<s^{*} \\ 1 & \text { otherwise }\end{cases}
$$

and

$$
G_{B}\left(e ; s^{*}\right)= \begin{cases}c(e+\Delta) & \text { if } e<\bar{b}-\Delta \\ c(\bar{b}) & \text { if } \bar{b}-\Delta \leq e<s^{*}-\Delta \\ 1 & \text { otherwise }\end{cases}
$$

, where $\bar{b}=c^{-1}\left(2 c\left(s^{*}\right)-1\right)$ and $w_{B}=1-2 c\left(s^{*}-\Delta\right)+c(\bar{b}-\Delta)$.

- When $\Delta<\Delta_{1}=2 c^{-1}\left(\frac{1}{2}\right)-1=\sqrt{\frac{2}{t}}-1$, both firms choose exactly $s^{*}: e_{A}=s^{*}$, $e_{B}=1-s^{*}$.
- When $\Delta \in\left[\Delta_{1}, \Delta_{2}\right]$, each firm randomizes between two points:

$$
\mathrm{P}\left(e_{A}=s^{*}\right)=2 c\left(s^{*}-\Delta\right)=1-\mathrm{P}\left(e_{A}=0\right)
$$

and

$$
\mathrm{P}\left(e_{B}=1-s^{*}\right)=2\left(1-c\left(s^{*}\right)\right)=1-\mathrm{P}\left(e_{B}=0\right)
$$

Lastly, I will use the four thresholds above to define what are high cost, moderate cost and low cost.

- $t<\frac{1}{2}$ : costs are low enough so that $\Delta_{1}>1$. In this case, no matter where $s^{*}$ is, players always move to exactly $s^{*}$.
- $\frac{1}{2} \leq t<1$ : costs are low, namely $\Delta_{2}>1>\Delta_{1}$. In this case, both firms either choose $s^{*}$ with probability one or randomize between their initial locations and $s^{*}$.
- $1 \leq t<2$ : most complicated case, namely moderate costs so that $\Delta_{4}<1$ and $\Delta_{2}>0$. In this case, all five possible cases in Figure 5 are possible.
- $2 \leq t<4$ : costs are high so that $\Delta_{2}<0$. In this case, firms never randomize between two points or choose $s^{*}$ with probability one.
- $t>4$ : costs are so high that $\Delta_{3}<0$ and bid cap is never hit for the weaker firm.


## G Proof of Proposition 6

When $t<\frac{1}{2}$, the maximum possible cost of adjustment is $t * 1^{2}<\frac{1}{2}$. Therefore, wherever $s^{*}$ is revealed to be, both firms have incentives to submit exactly $s^{*}$ as their design. As a result, the contest organizer always gets the first best result.

On the other extreme, for high cost $t>4$, revelation of $s^{*}$ leads to one of the following two scenarios: if $s^{*}$ is far away from the median, no firm puts any effort because it is too costly for the weaker firm to catch up; if $s^{*}$ is close to $\frac{1}{2}$, it is a case of contests with head start (high cost guarantees no bid cap in effect). Expected utility of the contest organizer is $U^{D}=\frac{23}{72 t}-\frac{1}{60 \sqrt{t}}-\frac{1}{4}$. On the other hand, the expected utility of the contest organizer if she conceals is $U^{C}=-\frac{1-2 t+2 t^{2}}{8 t^{2}}$. Their difference, $U^{D}-U^{C}=\frac{-6 t^{3 / 2}+25 t+45}{360 t^{2}}$ converges to zero from below as $t \rightarrow \infty$. In other words, the contest organizer is better off concealing information about $s^{*}$ when costs are high.

For $\frac{1}{2}<t<4$, it can be shown that $U^{D}-U^{C}>0$.

## H Instructions in the CH Treatment ${ }^{13}$

## Instructions

## Overview

You are about to participate in an experiment in decision making. Your decisions will determine your earnings. Please read these instructions carefully.

Do not use mobile phones, laptop computers, or use the lab computer for other purposes. During the experiment, please refrain from talking or looking at the computer monitors of others. If you have questions anytime, please raise your hand and we will address it as soon as possible. In the experiment, you will earn points. After the experiment, we will convert your points and pay you the Chinese Yuan at the rate of 100 points $=10$ Chinese Yuan.

Today's experiment consists of two parts, followed by a questionnaire. Your decisions made in each part only affect your earnings in that part.

## Part 1 Instructions

Part 1 of the experiment consists of 40 decision rounds. Before the first round, you will be randomly and anonymously grouped with another player in the room. You will play with this player for all 40 rounds in this part of the experiment. In each decision round, both you and the other paired player act as a shop owner and choose new locations of your shops. At the outset, the two shops are at the endpoints of a 100-meter-long street. Each of you has 150 points for the use of moving the shop to a new location on the street. For the same moving distance, the moving cost is the same for both of you. Moreover, the moving cost increases in the moving distance, and at a higher rate as distance increases. Specifically, the moving cost equals $8 \%$ of the square of the moving distance. For instance, if you choose to move your new shop 30 meters away from your old shop, the cost is $8 \% * 30 * 30=72$ points. In the experiment, as shown in Figure 1, you can move your mouse to check the moving

[^10]cost for each location on the street. Once you decided your most preferred location, you can confirm your choice by clicking the mouse.

Round 1 / 40


The new locations chosen by you and the other player will determine the relative attractiveness of two shops for a consumer. To be specific, a consumer ("she") arrives at a random location on the street and picks the shop that is closer to her. She will bring a profit of 100 points to the shop owner. In the case of a tie, the 100 points will be equally split between two players. Before choosing the new locations, you and the other player would know the location of the consumer. Please note that even if the consumer ends up going to the other shop, your moving cost will nevertheless be deducted from your balance. When you choose the moving location, you can also check your earnings for each possible scenario on the screen. See Figure 1.

Your earnings for each round equal to the starting balance minus your moving cost and then plus your share of profits from the consumer. If your shop is strictly closer to the consumer
than the other shop is, you will have a profit of 100 points. If two shops are equally close to the consumer, you will have a profit of 50 points. Alternatively, if your shop is further away to the consumer compared to the other shop, you will have 0 points. For instance, suppose at the beginning of the round, your old shop locates at 0 and the other player's shop locates at 100. You then spend 19.2 points in moving your shop to the location 15.5 and the other shop moves to the location 71.5. Suppose the consumer arrives at the location 90.4 , which is closer to the shop owned by the other player than to your shop. Your earnings in this round are $150-19.2=130.8$ points. Your net profit is your earnings minus your start balance, which is $130.8-150=-19.2$ in the above example. In the experiment, you can check both earnings and net profits on the result page or in the history table.

After you and the other player make the location choices, the screen will show the results of the current round. You can check the new locations of two shops and the earnings. After confirming the earnings, you will proceed to the next decision round. Each round is independent of other rounds. In a new round, you and your paired player will have a refreshing start balance of 150 points. A new consumer will arrive and bring a profit of 100 points. The consumer's location is known before choosing locations for the new shops.

At the end of this part of the experiment, 2 of the 40 rounds will be randomly selected for payment. Each round is equally likely to be chosen. Your earnings in the selected rounds will be counted to your final payment.

Part 2 Instructions In part 2, you will make 10 decisions. In each decision, you will choose between Option A and Option B. Each of these two options has a probability to generate either high earnings or low earnings. In all 10 decisions, high and low earnings in Option A are fixed at 40 points and 32 points, and high and low earnings in Option B are fixed at 77 points and 2 points. In each decision, Options A and B have the same probability to generate high earnings. For example, in the first decision, both options have $10 \%$ chance to generate high earnings.


You could click Option A or Option B buttons in blue to make your choices. After you finish all 10 choices, one of them will be randomly selected for payment. Computer will generate a random number, which determines whether your earnings are high or low (with the specified probability) Each decision is equally likely to be chosen. For example, the graph below shows a chosen round for payment. Because you have chosen Option B, and the random number determines that you will earn a high earnings, therefore your earnings in this part is 77 points.
Option A

[^11]
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[^0]:    ${ }^{1}$ Overbidding in my experiment means subjects make more adjustments than the equilibrium predicts.

[^1]:    ${ }^{2}$ I will use the terms "submissions" and "designs" interchangeably.

[^2]:    ${ }^{3}$ Exceptions to regular strategies are rare and artificial: for instance, Siegel (2010)'s online appendix provides an example in which cost functions are constructed with Cantor functions.

[^3]:    ${ }^{4}$ This commitment assumption is in line with the information design literature [see, for example, Kamenica and Gentzkow (2011)].

[^4]:    ${ }^{5}$ Overbid in this context means that subjects adjust more than the equilibrium predicts.

[^5]:    ${ }^{6}$ At the threshold costs predicted by the theory, maximum affordable moving distance is 2.7 and equilibrium moving distance is 1.25 . Recall that the length of the street is 100 .
    ${ }^{7}$ This corresponds to $t=1.25$ and $t=8$ in the model.

[^6]:    ${ }^{8}$ As a reference, Beijing and Shanghai, two of the biggest cities in China, have minimum hourly wages of $¥ 24$ and $¥ 22$ in 2019 .

[^7]:    ${ }^{9}$ The threshold distance is 50 in the DH treatment and 67.68 in the DL treatment.

[^8]:    ${ }^{10}$ In the disclose treatments, when the equilibrium is mixed strategies, the benchmark is the mean of the equilibrim strategy.
    ${ }^{11}$ Recall that equilibrium prediction is 3.125 in the CH treatment and 20 in the CL treatment.

[^9]:    ${ }^{12}$ In mathematics, the resultant of two polynomials is a polynomial expression of their coefficients, which is equal to zero if and only if the polynomials have a common root.

[^10]:    ${ }^{13}$ These are translated from the Chinese version used in the experiment. Instructions in Chinese are available upon request.

[^11]:    As in the graph above, your choice was B. According to the random number, the green result is realized for your payment.

    Hence, your earnings in Part $\mathbf{2}$ is $\mathbf{7 7 . 0}$ pts.

